Differential Geometry (M24)

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This course is intended as an introduction to modern differential geometry. It can be taken with a view of further studies in Geometry and Topology and should also be suitable as a supplementary course if your main interests are e.g. in Analysis or Mathematical Physics. A tentative syllabus is as follows.

- *Local Analysis and Differential Manifolds.* Definition and examples of manifolds, matrix Lie groups. Tangent vectors, the tangent and cotangent bundle. Geometric consequences of the implicit function theorem, submanifolds. Exterior algebra of differential forms. Orientability of manifolds. Partition of unity and integration on manifolds, Stokes' Theorem. De Rham cohomology.
- *Vector Bundles.* Structure group, principal bundles. The example of Hopf bundle. Bundle morphisms. Three views on connections: vertical and horizontal subspaces, Christoffel symbols, covariant derivative. The curvature form and second Bianchi identity.
- *Riemannian Geometry.* Connections on manifolds, torsion. Riemannian metrics, the Levi–Civita connection. Geodesics, the exponential map, Gauss' Lemma. Decomposition of the curvature of a Riemannian manifold, Ricci and scalar curvature, low-dimensional examples. The Hodge star and Laplace–Beltrami operator. Statement of the Hodge decomposition theorem (with a sketch-proof, time permitting).

Pre-requisites

An essential pre-requisite is a working knowledge of linear algebra (including dual vector spaces and bilinear forms) and of multivariate calculus (e.g. differentiation in several variables and the inverse function theorem). Exposure to some ideas of classical differential geometry would be useful.

Literature

[1] R.W.R. Darling, *Differential forms and connections.* CUP, 1994.

[2] S. Gallot, D. Hulin, J. Lafontaine, *Riemannian geometry.* Springer-Verlag, 1990.

[3] V. Guillemin, A. Pollack, *Differential topology.* Prentice-Hall Inc., 1974.

[4] F.W. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer, 1983.

Roughly, half of the course material is taken from [4]. The book [3] covers the required topology. On the other hand, [1] which has a chapter on vector bundles and on connections assumes no knowledge of topology. Both [1] and [2] have a lot of worked examples. There are many other good differential geometry texts, e.g. the five volume series by M. Spivak.

Additional support

The lectures will be supplemented by four example classes, the fourth class to take place at the beginning of Lent Term will also fulfill a revision function. Printed notes will be available from https://www.dpmms.cam.ac.uk/~agk22/teaching.html

DIFFERENTIAL GEOMETRY. Series of Lecture Notes and Workbooks for Teaching Undergraduate Mathematics. AlgoritmuselmÂ´elet Algoritmusok bonyolultsÂ´aga Analitikus mÂ´odszerek a pÂ´enzuÂ¨gyekben BevezetÂ´es az analÂ´Ä±zisbe Dierential Geometry DiszkrÂ´et optimalizÂ´alÂ´as DiszkrÂ´et matematikai feladatok Geometria IgazsÂ´agos elosztÂ´asok InteraktÂ´Ä±v analÂ´Ä±zis feladatgyuËjtemÂ´eny matematika BSc hallgatÂ´ok szÂ´amÂ´ara Introductory Course in Analysis Matematikai pÂ´enzuÂ¨gy Mathematical Analysis-Exercises 1-2 MÂ´ertÂ´ekelmÂ´.Â SUMMARY: The aim of this textbook is to give an introduction to dier-ential geometry. It is based on the lectures given by the author at EÂ¨otvÂ¨os LorÂ´and University and at Budapest Semesters in Mathematics. Geometry and Topology. Algebraic Topology (M24). Differential Geometry (M24). Mapping class groups (M16). Symplectic Topology (L24).Â Topics in Convex Optimisation (M16). Numerical Solution of Differential Equations (L24). Hybrid Photonics Computing (L16). Mathematical Analysis of the Incompressible Navier-Stokes Equations (L24) â€" Nonâ€"Examinable. Euclidean Spaces. Introduction to differential geometry. Joel W. Robbin UW Madison. Dietmar A. Salamon ETH ZuÂ¨rich.Â One can distinguish extrinsic dierential geometry and intrinsic dier-ential geometry. The former restricts attention to submanifolds of Euclidean space while the latter studies manifolds equipped with a Riemannian metric. The extrinsic theory is more accessible because we can visualize curves and surfaces in R3, but some topics can best be handled with the intrinsic theory. The denitions in Chapter 2 have been worded in such a way that it is easy to read them either extrinsically or intrinsically and the subsequent chapters are mostly (but not entirely) extrinsic.