

The content of these works, mathematically very complex, is impossible to describe in a few words. I shall limit myself to some general remarks concerning the methods of proof. The expression “infinitesimal mathematics” in the general title of the volumes should not be taken literally: in none of the works published is there explicit mention, let alone a technical use, either of infinitely small magnitudes or of indivisibles in the sense of Cavalieri. The term “infinitesimal” refers, rather than to the methods of proof, to the content of the various works, in their majority dedicated to the Archimedean problems of the calculations of areas and volumes of figures. The proofs are also in Archimedean style, being based on the classical method of exhaustion, that all the authors handle with great technical skill. If a difference exists with respect to the classical proofs, it is rather found in the preference of the Arabic authors for “numerical” calculations rather than for the theory of proportions. For instance, we read in the text of the Banū Mūsā that the area of the triangle of base b and height h is given by $(1/2)bh$, a statement that would hardly be found in a classical author.

But this is matter for a research project rather than for a review, and such a project, like many others, would never have been possible without having access to the texts in Rashed’s edition. All who now wish to comment on Arabic contributions to infinitesimal mathematics will have to study with care the works published in these volumes.

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La découverte de la géométrie non euclidienne sur la pseudosphère: Les lettres d’Eugenio Beltrami à Jules Hoüel (1868–1881). Introduction, notes and critical commentaries by Luciano Boi, Livia Giacardi, Rossana Tazzioli. Preface by Christian Houzel and Eberhard Knobloch. Collection sciences dans l’histoire. Paris (Librairie Scientifique et Technique Albert Blanchard). 1998. 278 pp. + iii. 480 FF.

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This rich book illuminates a number of topics connected with the successful diffusion of non-Euclidean geometry. It is well known that the new geometry was discovered independently by Janos Bolyai in Hungary and Nicolai Ivanovich Lobachevskii in Russia, but that their publications fell dead from the press. Carl Friedrich Gauss worked much of the matter out for himself before their work reached him, but he never published what he knew, and Bernhard Riemann’s fundamental contribution to the reformulation of geometry was also published only posthumously, in 1868. By then the stage was set for a new and more convincing presentation, and this was provided mathematically by Eugenio Beltrami in his famous *Saggio* of 1868, while in 1866 and 1867 the energetic Jules Hoüel saw to it that the works of Bolyai, Lobachevskii, Beltrami, and others were republished in French and henceforth accessible.

Beltrami’s letters to Hoüel take us, as it were, behind the scene. They show very clearly how Beltrami accepted the differential geometric foundations of geometry, which he

attributed largely to Gauss, in place of axiomatic foundations in the manner of Euclid. They also show that they took their advocacy of non-Euclidean geometry very seriously, for in the early days there were a number of mathematicians who still refused to accept it. The celebrated blunder of Joseph Bertrand in 1869, who published a fallacious defence of the parallel postulate due to Jules Carton, and was accordingly criticized by Joseph Liouville and Irénée-Jules Bienaymé (and supported by Charles Dupin), is indicative of the ingrained hold of Euclidean geometry on mathematicians of the day. Beltrami was content that Liouville take up the cause in France. It is one of the pleasures of such editions as this that we learn that, in Beltrami's opinion at least (see letter 19), "Liouville has a very decided antipathy to Italy and the Italians ...It seems that he nourishes this aversion as a compensation for his double love of the republic and the pope."

Beltrami has comments on the mathematics and on the precise wording of the translations. He notes, for example (letter 17), that his pseudo-spherical model is a good description of two-dimensional non-Euclidean geometry, but there is no analogous description of three-dimensional non-Euclidean geometry in ordinary Euclidean space. Problems with the three-dimensional version bedeviled most people who worked on the topic, and the Euclidean rearguard could exploit this. In November 1868 (see letter 1) even Beltrami was prepared to regard Lobachevskii's ideas on three-dimensional non-Euclidean geometry as a geometric hallucination, and Luigi Cremona had his doubts.

The attachment of the new generation of Italian mathematicians to non-Euclidean geometry is interesting, and this book shows clearly who stood where on the issue. Guiseppe Battaglini was in favour, Angelo Genocchi less certain (see letter 37). His objection was based on his belief that a surface was necessarily embedded in a three-dimensional space; Beltrami argued against this by citing not Riemann, as we would, but Gauss, which may show the source of his inspiration.

The success of the new geometry eventually became apparent. Beltrami was drawn into correspondence with others about it, and some of those letters are usefully reprinted here (with Felix Klein and Hermann von Helmholtz) along with letters between other mathematicians. The correspondence moved on, with less vigor, to other topics: from the technically difficult study of differential parameters to the ongoing controversy (in England and Italy) about the merits of teaching Euclid's *Elements* in schools. Hoüel (see p. 56) was in favor provided the ancient text was explained, commented upon, and even corrected by a competent teacher, but not if it was to be learned by rote as the English required. One scarcely wonders which country was to produce more research mathematicians over the next 30 years. The editors are to be congratulated on providing a lucid introduction to Beltrami and his work and for bringing Hoüel onto center stage. His own journey, as a good but not a profound mathematician, from confidence in Euclidean to proselytizing for non-Euclidean geometry is well traced, and one notes that it precedes the publication of Beltrami's *Saggio*. The letters are supported by footnotes which are helpful and informative and do not take over the page, and a wide range of relevant primary and secondary literature is well deployed. The letters between Darboux and Hoüel were once edited by Hélène Gispert [1], and one might hope that they too could be republished, because they cover some of the same ground and further illuminate the importance of the journal Darboux and Hoüel founded, the *Bulletin des sciences mathématiques et astronomiques*. But I should like to end with a piece of trivia that underlines the value of editions such as this one.

Beltrami went to Venice in 1871 to help his wife recover from some sort of crisis, and wrote to Hoüel (letter 37) that “I never cease to be pained by the fatal indifference of the new generation to anything that is serious and a little difficult.” Plus ça change? Beltrami was then 35.

REFERENCE

1. Hélène Gispert, La correspondance de G. Darboux avec J. Hoüel: Chronique d'un rédacteur (déc. 1869–nov. 1871), *Cahiers du Séminaire d'histoire des mathématiques* 8 (1987), 67–202.

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The Topkapi Scroll—Geometry and Ornament in Islamic Architecture: Topkapi Palace Library MS. H. 1956; With an Essay on the Geometry of the Muqarnas by Mohammad al-Asad. By Gülru Necipoğlu. Santa Monica (The Getty Center for the History of Art and the Humanities). 1995, 395 pp.

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This is a magnificent book, and a very scholarly book as well. First, it is a joy to admire the reproductions of the scroll; reading the accompanying text, one is captured by the explanations and information which treat much more than only the scroll. What is it all about? The scroll, preserved at the Topkapi Palace Museum Library and reproduced in its entirety in this volume, is a pattern book from the workshop of a master builder. It was probably compiled in the late 15th or 16th century somewhere in western or central Iran, probably in Tabriz, a city which once rivaled the Timurid capitals of Samarqand and Herat in architectural splendor. Because of the nearly complete destruction of monuments in Tabriz, the scroll's patterns might be seen as precious mementos of a now-lost archaeological record. How the scroll found its way into the Ottoman imperial treasury collection is not easy to answer. The Ottomans conquered Tabriz several times, on which occasions many skilled artisans and scholars as well as treasures and manuscripts were taken from Tabriz to Istanbul. The scroll could also have been brought to Istanbul by such personages as the Timurid astronomer–mathematician Ali Kuşci, who joined the court at Istanbul in 1472 with a large manuscript collection including mathematical treatises. Another possibility is that it belonged to Timurid–Turkmen decorators and workmen who were invited to the court in the 1470s. If the Topkapi scroll had been used in the office of the chief architect, it would probably have disappeared together with all the architectural drawings cited in the Ottoman sources. Having been deposited in the Inner Treasury of the Topkapi Palace and then forgotten, thanks to its minor relevance to classical Ottoman architecture, it is preserved in superb condition. The implications of the scroll for architectural practice in the late Timurid–Turkmen and early Safavid worlds are of great importance. The Topkapi scroll is a unique document.

Necipoğlu's book has won an award from the U.S. Society of Architectural Historians and was named the best new scholarly book in the field of architecture and urban planning

La lettre informelle. Seville, le 21 novembre 2010 Chre Beatrice, Je voudrais te remercier pour la formidable semaine que j'ai passe chez toi, Paris. Je ne me suis pas ennuye une seconde, il y a tant de choses voir et a faire ! J'ai t trs impressionne par Paris, et puis bien sur, passer le quatorze juillet dans une famille franaise, cest toute une exprience ! Il faudra que tu remercies ta famille de ma part, ils ont tous t trs gentils et trs disponibles. Jespre que tu tiendras parole et que tu viendras Seville lete prochain. Eugenio Beltrami (16 November 1835 – 18 February 1900) was an Italian mathematician notable for his work concerning differential geometry and mathematical physics. His work was noted especially for clarity of exposition. He was the first to prove consistency of non-Euclidean geometry by modeling it on a surface of constant curvature, the pseudosphere, and in the interior of an n-dimensional unit sphere, the so-called Beltrami–Klein model. He also developed singular value decomposition for matrices la gÃ©omÃ©trie non euclidienne - Noun gÃ©omÃ©trie non euclidienne / gÃ©omÃ©tries non euclidiennes , f - nicht euklidische Geometrie. Singulier Masculin. la gÃ©omÃ©trie non euclidienne. Singulier FÃ©minin. les gÃ©omÃ©tries non euclidiennes. Translated from French into German by. Popular English Verbs. In this section you can decline nouns and adjectives, conjugate verbs in all modes and tenses in 5 languages - English, German, French, Spanish and Russian. Learn the rules of conjugation and declension and see translations in context.