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# THEORY AND PROBLEMS

OF

# DIFFERENTIAL EQUATIONS IN SI METRIC UNITS

FIRST EDITION

BY

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# Contents

CHAPTER		PAGE
Chapter 1	ORIGIN OF DIFFERENTIAL EQUATIONS . . . . .	1
Chapter 2	SOLUTIONS OF DIFFERENTIAL EQUATIONS . . . . .	7
Chapter 3	EQUATIONS OF FIRST ORDER AND FIRST DEGREE . . . . .	12
Chapter 4	EQUATIONS OF FIRST ORDER AND FIRST DEGREE —VARIABLES SEPARABLE AND REDUCTION TO VARIABLES SEPARABLE . . . . .	15
Chapter 5	EQUATIONS OF FIRST ORDER AND FIRST DEGREE —EXACT EQUATIONS AND REDUCTION TO EXACT EQUATIONS . . . . .	24
Chapter 6	EQUATIONS OF FIRST ORDER AND FIRST DEGREE —LINEAR EQUATIONS AND THOSE REDUCIBLE TO THAT FORM . . . . .	35
Chapter 7	GEOMETRIC APPLICATIONS . . . . .	41
Chapter 8	PHYSICAL APPLICATIONS . . . . .	49
Chapter 9	EQUATIONS OF FIRST ORDER AND HIGHER DEGREE . . . . .	61
Chapter 10	SINGULAR SOLUTIONS—EXTRANEOUS LOCI . . . . .	67
Chapter 11	APPLICATIONS OF FIRST ORDER AND HIGHER DEGREE EQUATIONS . . . . .	75
Chapter 12	LINEAR EQUATIONS OF ORDER $n$ . . . . .	78
Chapter 13	HOMOGENEOUS LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS . . . . .	82
Chapter 14	LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS . . . . .	87
Chapter 15	LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS —VARIATION OF PARA- METERS, UNDETERMINED COEFFICIENTS . . . . .	93
Chapter 16	LINEAR EQUATIONS WITH CONSTANT COEFFICIENTS —SHORT METHODS . . . . .	99
Chapter 17	LINEAR EQUATIONS WITH VARIABLE COEFFICIENTS —THE CAUCHY AND LE- GENDRE LINEAR EQUATIONS . . . . .	108
Chapter 18	LINEAR EQUATIONS WITH VARIABLE COEFFICIENTS —EQUATIONS OF THE SECOND ORDER . . . . .	111
Chapter 19	LINEAR EQUATIONS WITH VARIABLE COEFFICIENTS —MISCELLANEOUS TYPES . . . . .	122
Chapter 20	APPLICATIONS OF LINEAR EQUATIONS . . . . .	133
Chapter 21	SYSTEMS OF SIMULTANEOUS LINEAR EQUATIONS . . . . .	157
Chapter 22	TOTAL DIFFERENTIAL EQUATIONS . . . . .	164
Chapter 23	APPLICATIONS OF TOTAL AND SIMULTANEOUS EQUATIONS . . . . .	178
Chapter 24	NUMERICAL APPROXIMATIONS TO SOLUTIONS . . . . .	186



## Index

- Acceleration, 49  
Amplitude, 134  
Applications,  
  of first order and higher degree equations, 75-77  
  geometric, 41-48, 75-77, 133, 136, 178  
    polar coordinates, 42, 46  
    rectangular coordinates, 41, 44, 45  
    trajectories, 43, 47, 48  
  of linear equations, 133-156  
    electric circuits, 136, 151-154  
    geometric, 133, 136  
    horizontal beams, 134-136, 145-151  
    oscillatory motion, 133, 134, 137, 138, 140-143  
  of total and simultaneous equations, 178-185  
  physical, 49-60, 133-156, 178-185  
    electric circuits, 57, 58, 136, 151-154, 183  
    hanging cable, 144  
    horizontal beam, 134-136, 145-151  
    motion along a straight line, 49, 54-56, 138  
    motion of a complex system, 139  
    motion of a pendulum, 137  
    oscillatory motion, 133, 134  
    springs, 140-143  
Approximation,  
  numerical, 186-196  
Arbitrary constant, 1, 78, 231, 232, 234  
Arbitrary function, 232-236  
Auxiliary system, 239  
  
Beams,  
  horizontal, 134-136, 145-151  
Bernoulli's equation, 35, 37  
Bessel equation, 222, 227, 228  
Bessel functions, 222, 228  
  
Cauchy linear equation, 108, 109, 113  
Cauchy (Ordinary) Differential Equation, 269  
C-discriminant, 69-74  
Characteristic equation  
  complex roots, 83, 85  
  distinct real roots, 83, 84  
  repeated roots, 83, 85  
Characteristic Roots, 83, 85  
Charpit's method, 247, 253  
Clairaut equation, 62, 64, 65, 71, 75, 76, 124  
Complementary function, 79, 257-259, 261-263, 266, 267, 269-272  
Complete differential, 12  
Complete solution,  
  of ordinary differential equation, 79  
  of partial differential equation, 240, 244-253  
Conditions,  
  for exactness, 24, 165  
  for integrability, 164, 167-170  
  for linear independence, 78, 80, 81  
  
Damping factor, 134  
Degree, of differential equation, 1, 61  
Derivatives,  
  ordinary, 1  
  partial, 1  
  
Differential equation,  
  Bernoulli, 35, 37  
  Bessel, 222, 227, 228  
  Clairaut (see Clairaut equation)  
    62, 64, 65, 71, 75, 76, 124  
  conditions for solubility of, 7  
  definition, 1  
  exact, 12, 24-34, 123, 129-131  
  extended Clairaut, 234, 246  
  first order, first degree, 12-40  
    exact equations, 12, 24-34  
    homogeneous equations, 15-18  
    linear equations, 35-40  
    linear but not homogeneous, 16, 19, 20  
    miscellaneous substitutions, 16, 21  
    variables separable, 13, 15-23  
  first order, higher degree, 61-66  
  first order, simultaneous, 189, 193, 194  
  Gauss, 223, 229  
  homogeneous ordinary, 15, 17, 18, 78, 82-86  
  homogeneous partial, 255-264  
  Legendre, 220, 221, 224-226  
  Legendre linear, 108-110  
  linear with constant coefficients, 87-107  
    short methods, 99-107  
    systems of, 157-163  
    undetermined coefficients, 93, 96-98  
    variation of Parameters, 93-96  
  linear,  
    homogeneous with constant coefficients,  
      82-86  
    or order one, 13, 35-40  
    of order  $n$ , 78-81, 122-132  
    of order two, 111-121, 199, 202-205  
    partial, first order, 238-243, 277  
    partial, higher order, 276-293  
    with variable coefficients, 108-132  
  non-homogeneous, partial, irreducible,  
    268, 269, 272, 273  
  non-homogeneous, partial, reducible,  
    265-268, 270-272  
  non-linear partial, 238, 244-254, 280  
  numerical solution, 186-196  
  ordinary, 1, 157  
  origin of, 1-6  
  partial 1, 157, 231-293  
    order of, 231  
  partial, first order, 238-254  
  partial, higher order,  
    constant coefficients, 255-275  
    variable coefficients, 276-293  
  solutions of, 7-11  
  solutions in series, 197-211  
  systems of, 157-163  
  total, 164-177  
Direction field, 8  
Discriminant, 69-74  
Discriminant relation, 69, 70  
D-Notation, 82  
  
Electric circuits, 57, 58, 136, 151-154, 183  
Exact differential, 24, 25

- Exact equations, 12, 24–34, 123, 129–131  
 reduction to, 24
- Existence theorem, 7
- Extended Clairaut equation, 234, 246
- Extraneous equation, 68
- Extraneous loci, 67–74
- First derivative method, 187, 191
- Force, 49
- Frequency, 134
- Functions,  
 Bessel, 222, 228  
 complementary, 79, 257, 266  
 homogeneous, 15
- Gauss equation, 223, 229
- General solution,  
 of ordinary differential equation, 7  
 of partial differential equation,  
 238–242, 244, 245, 256, 257, 265–267
- Harmonic motion, 133, 134, 137, 138, 140–143
- Homogeneous equation, 15–18
- Homogeneous function, 15
- Homogeneous linear equation, 78–86, 255–264
- Hooke's law, 55
- Hypergeometric series, 223
- Indicial equation, 208, 210, 212  
 Roots Differing by an Integer, 212  
 Roots Equal, 210
- Infinite series, 197
- Integral curve, 7–9, 41, 43
- Integration factor, 12, 24
- Integration in Series, 197–205, 206–219
- Intermediate integral, 280
- Irreducible equation, 265, 268, 269, 272, 273
- Kutta's Simpson's method, 188, 193, 195
- Lagrange system, 239
- Laplace's transformation, 278, 279, 286, 287, 292
- Large values of  $x$ , 208, 216
- Legendre equation, 220, 221, 224–226
- Legendre linear equation, 108–110
- Legendre polynomial, 221, 226
- Linear equation (*see also* Differential equation,  
 linear) with Variable Coefficients:  
 Cauchy & Legendre Linear Equations,  
 108–110  
 Equations of the second order, 111–121  
 exact equations, 123, 129–132  
 dependent variable absent, 122, 124  
 independent variable absent, 122, 125, 126  
 particular integral known, 123, 126–128
- Loci, extraneous, 68
- Locus,  
 cusp, 69, 73  
 nodal, 69, 70, 72  
 tac, 69, 72
- Mass, 49
- Monge's Equations, 281, 282, 288, 289
- Monge's method, 288, 292
- Newton's law of cooling, 51
- Newton's second law of motion, 49
- Non-homogeneous linear equation, 16, 19
- Non-homogeneous linear partial differential  
 equation,  
 reducible, 265–268, 270–272  
 irreducible, 268, 269, 272, 273  
 with Constant Coefficients, 265–275
- Non-linear partial differential equation,  
 244–254, 280
- Numerical Approximations to Solutions, 186–195
- Operators, factorization of, 112
- Order,  
 of differential equation, 1, 5  
 reduction of, 122
- Origin,  
 of ordinary differential equation, 1–6  
 of partial differential equation, 231
- Orthogonal trajectories, 43, 47, 48
- Parameters, variation of, 93, 94
- Partial differential equations, 1, 231–237
- Partial fractions, method of, 88
- Particular Integral, 79, 257, 266
- Particular integral curve, 41
- Particular solution, 7, 9, 11, 79
- $p$ -discriminant, 69–74
- Period, 134
- Picard's method, 186, 189, 190, 193
- Point,  
 ordinary, 199  
 regular singular, 206  
 singular, 199, 206
- Primitive, 1–4
- Recursion formula, 198
- Reduction of order, 122
- Rodrigues' Formula, 224, 225
- Runge's method, 188, 192, 194
- Separation of variable, 13, 15–23
- Series,  
 hypergeometric, 223  
 solution in, 197  
 Taylor, 187, 191, 194
- Short methods,  
 ordinary differential equation, 99–107  
 partial differential equation, 266
- Simultaneous equations, 157–163
- Singular solution, 7, 67–74, 244
- Solutions,  
 complete, 79, 240, 244  
 general, 7, 11, 238, 242, 244  
 linearly independent, 78  
 particular, 7  
 in series, 197  
 singular, 7, 64–74, 244
- Springs, 140
- Steady-state phenomenon, 134
- S.I. System, 49
- System of equations, 157–163
- Taylor series, 187, 190, 191, 194
- Total differential equation, 164–177
- Trajectories, 43
- Transient phenomenon, 134
- Undetermined coefficients, method of,  
 93, 96, 98, 258
- Variables separable, 13, 15–23
- Variation of parameters, 93, 94, 98

“Systematicity” is the most capacious concept applicable to any object of research, which represents all the possible manifestations of systems. Law is a systemic phenomenon, characterized by an organized structure, which is characteristic of a great number of constituent elements and presence of different levels of functional connections between them. "Modern Methods, Problems and Applications of Operator Theory and Harmonic Analysis X". Rostov-on-Don, 22 - 27 AUGUST, 2021. The conference is dedicated to the 80 annual jubilee of professor Stefan G. Samko (Russia, Portugal). Working languages: English, Russian. Arrival: August 22nd. Departure: August 27th. Working days of the conference: 23 - 26 of August 2021. Deadline for registration and for abstracts submission: 01 August 2021. Theory and problems of social psychology. Item Preview. > remove-circle. Share or Embed This Item. EMBED. Theory and problems of social psychology. by. Krech, David. Publication date. 1948. Topics. Social psychology. Start studying Theory and problems test 1. Learn vocabulary, terms and more with flashcards, games and other study tools. Which of the following organizations is responsible for setting international financial reporting standards. D. International accounting standards board. According to the IASB framework for the preparations and presentation of financial statements, the fundamental qualitative characteristic of relevance includes. C .Predictive value and confirmatory value. Based on the problematic issues of a general theoretical nature, students’ skills of considering controversial issues and validity of their position are formed. Instructor: Credit points: 3 Faculty: Faculty of Law Language: Russian Level: Specialist Academic hours: 36. Syllabus Section I. 1. Actual problems of subject and method of the Theory of State and Law. Section II. 2. Problems of the origin and evolution of the state and law. Section III. 3. Problems of the definition, identifying key features and nature of the state.