

BOOK REVIEWS

(Wright, Welch, and Jollay), were described for a variety of polyphase materials.

Section IV (*Failure Analysis*) includes papers which focus upon criteria for predicting strength, fracture and/or failure of composite materials. Specifically, studies on strength criteria (Anin and Baev), failure of thin-walled structures under flexure (Nemirovsky), optimum design and strength (Obraztsov and Vasil'ev), fracture models (Rikards, Teters, and Upitis), influence of failure peculiarities on strength (Perov, Skudra, Mashinskaja, and Bulavs), free edge induced failure analysis (Crossman), bone fracture (Knets), and fatigue life prediction (Parfeyev, Oldirev, Tamuzs) are presented.

Section V (*Experimental Methods*) contains papers on the experimentally behaviors in composite materials and various techniques for observing and testing them. Included are nondestructive study of damage (Latishenko, Matiss), test method development (Chamis), the nature of crack growth (Bunsell), optical methods (Rowlands and Stone), effect of high modulus fibers (Kalnin), interesting mechanical behaviors (Chiao), fracture characteristics (Lachman), and fracture initiation prediction (Mast, et al.).

The foregoing studies include both unidirectionally reinforced and crossply laminates and covered both common (glass-epoxy) and advanced (graphite-aluminum, etc.) materials as well as some more exotic (asbestos cement, bone tissue) of the polyphase materials.

Taken collectively, this volume presents a summary of current approaches and considerations involved in developing predictions of the failure by fracture and its associated mechanisms of a fairly wide variety of polyphase materials. Limitations and restrictions of the theories are noted and experimental methods are discussed and used to obtain results for comparison with analytical predictions. The volume should provide a window for viewing a "state of the art" in composite fracture (a developing but incomplete discipline) as seen collectively by the contributors. As such, it should be found of interest to workers in both composite materials and fracture mechanics.

Three-Dimensional Problems of the Mathematical Theory of Elasticity and Thermoelasticity. By V. D. Kupradze, T. G. Gegelia, M. O. Bacheleishvili, and T. V. Burchuladze. Edited by V. D. Kupradze. North-Holland Publishing Co., New York. 1979. Pages 929. Price \$158.50.

REVIEWED BY E. STERNBERG¹⁰

This voluminous tome is a translation into English of the 1976 second edition of a monograph originally published by the Tbilisi University Press in 1968. Although the individual contributions of the four authors involved are not identified, it is safe to surmise that the work of Kupradze, who also served as editor of the book, was predominant in determining its scope and character.

The present treatise is chiefly concerned with the classical linearized theory of homogeneous and isotropic elastic solids, elastostatic and elastodynamic considerations being given more or less equal attention. Further, a substantial amount of space is devoted to a linearized version of couple-stress theory for perfectly elastic, centrosymmetric-isotropic materials, as well as to linear thermoelasticity theory.

Notwithstanding its title, the book places relatively little emphasis on the treatment of specific physically important problems. Instead, the authors are heavily preoccupied with uniqueness and existence issues, and spend a major part part of their effort on the characterization of the relevant problem classes in terms of singular integral equations. Some background for this approach is supplied in pre-

liminary chapters on basic singular solutions of the governing field equations, on the theory of singular integral equations, and on pertinent aspects of potential theory. Special mention should also be made of the three closing chapters, which pertain to contact problems for elastic media with inclusions, the use of generalized Fourier series, as well as to certain series and quadrature representations of solutions to half-space and quarter-space problems.

As ought to be apparent from the preceding all too cursory description, this is a rather unconventional treatise on elasticity theory, the choice of topics covered reflecting strongly the taste and bias of its authors. In particular, some readers—including the reviewer—may question whether couple-stress theory merits the emphasis it receives here.

There is also cause to wonder whether the authors have consistently achieved "the modern level of mathematical rigor" avowed in their preface. Indeed, the mathematical erudition affected in these pages is not always matched by an equal measure of conceptual clarity or genuine mathematical care.

A few examples drawn from the opening chapter on "Basic Concepts and Axiomatization" may serve to illustrate such misgivings. Here ordinary and couple-stresses are introduced (prior to any discussion of kinematics) through limit-definitions that are not made mathematically meaningful. The repeated allusions to molecules or particles seem neither helpful nor appropriate in the context of a continuum-mechanical exposition. In view of the authors' casualness in distinguishing material from spatial coordinates, their transition to the linearized theory will not bear scrutiny. Nor is the reader aided by the admonition (on p. 17) not to confuse the "vector of rigid rotation" with the "vector of internal rotation," despite the use of two different symbols, since both are defined as one-half the displacement-curl (see pp. 9, 16).

No credit is given to the translator of this volume, and not much credit is due in this connection. Sentences such as "One may have an infinite number of directions at each point of a medium" (p. 5), are apt to be attributable to faulty translation. So is the puzzling assertion (p. 2): "If the body . . . is deformed, . . . the parts of the body are no longer in mechanical equilibrium."

While this is hardly a treatise suitable for uninitiated students of elasticity theory, it renders accessible in English some valuable material of interest to specialists in this subject area.

Theoretical Kinematics. By D. Bottema and B. Roth. North-Holland Publishing Company, Amsterdam, New York, Oxford. 1979. Pages 558 + XIV. Price \$87.75.

REVIEWED BY G. R. VELDKAMP¹¹

This is a textbook in which kinematics is presented as a theory independent of any particular application, that is: as a fundamental science in its own right. The bulk of the book, 420 pages, is devoted to Euclidean kinematics of 3 and 2 dimensions. A characteristic feature of the treatment is the principle of starting with general concepts and problems and then specializing gradually to more simple cases. The first 22 pages of the text are accordingly written in terms of n -dimensional Euclidean space. The whole matter is in the main treated analytical accompanied by ample geometric interpretation. Synthetic reasoning however is not evaded in those places where it may contribute to a deeper understanding of the problem on hand. The mathematical tools are borrowed from elementary algebraic geometry, calculus, vector, and matrix algebra; mathematical concepts which

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11 Anisotropic Elasticity. 12 Thermoelasticity. 13 Displacement Potentials and Stress Functions. 14 Nonhomogeneous Elasticity. PREFACE xi. Elasticity theory establishes a mathematical model of the deformation problem, and this requires mathematical knowledge to understand formulation and solution procedures. Governing partial differential equations are developed using basic principles of continuum mechanics commonly formulated in vector and tensor language. Elasticity theory is formulated in terms of many different types of variables that are either specified or sought at spatial points in the body under study. Some of these variables are scalar quantities, representing a single magnitude at each point in space. Elasticity Theory. The central model of solid mechanics. Rubber, metals (and alloys), rock, wood, bone can all be modelled as elastic materials, even though their chemical compositions are very different. This is the deep problem of the approach to equilibrium, having its origins in the Second Law of Thermodynamics. We will see how rather generally the balance of energy plus a statement of the Second Law lead to the existence of a Lyapunov function for the governing equations. Balance of Energy. For thermoelasticity, $W(F, \hat{I}_j)$ can be identified with the Helmholtz free energy $U(F, \hat{I}_j) = \hat{I}_j \cdot (F, \hat{I}_j)$. Hence, if the dynamics and boundary conditions are such that as $t \rightarrow \infty$ we have $\dot{y} \rightarrow 0$ and $\hat{I}_j \rightarrow \hat{I}_{j,0}$, then this is close to saying that y tends to a local minimizer of $\hat{I}_{j,0}(y) = [W]$. Thermoelasticity. Constitutive Equations. Plane Stress. Materials with one Plane of Symmetry. Three Orthogonal Symmetry Planes. Orthotropy. Transverse Isotropy. Strain Energy. Two-Dimensional Theory of Elasticity. Plane Stress. Plane Displacements. Problems. Theory of Elasticity. Introduction. The Hookean Solid. An Alternative Development of the Generalized Hooke's Law. Strain Energy. Two-Dimensional Theory of Elasticity. Plane Stress. Plane Displacements. three-dimensional boundary value problem of statics of the theory of elastic mixtures with given displacements on the boundary splits in the general case. Equations (12) and (13) can be combined as one equation $\hat{\tau} + M \text{grad div } v = 0$, (14) where $v = v_0$ for $M = M_1$ and $v = v_{00}$ for $M = M_2$. 3. V. D. Kupradze, T. G. Gegelia, M. O. Bacheleishvili, and T. V. Burchuladze, Three-dimensional problems of the mathematical theory of elasticity and thermoelasticity. (Translated from Russian) North-Holland series in applied mathematics and mechanics, v. 25, North-Holland Publishing Company, Amsterdam-New York-Oxford, 1979; Russian original: Nauka, Moscow, 1976.