Meeting the Demands of Calculus AND College Life: The Mathematical Experiences of Graduates of Some Reform-Based High School Programs

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Paper prepared for the Navigating Mathematical Transitions Symposium
session #39.11
2001 Annual Meeting of the American Educational Research Association
Seattle, WA. April 2001
(Discussant: Jim Fey)

The research reported in this paper was supported by a grant from the National Science Foundation (REC-9903264). The views expressed here do not necessarily represent the views of the Foundation.

Introduction and Overview

In this era of curricular and pedagogical reform in K–12 mathematics education, the entry to college and collegiate mathematics is a focus and concern for all—students, parents, university and high school educators, and administrators. Perhaps no other educational "transitions" matters as much to so many as the entry to and first year in college. While students may enter (and re-enter) college at any age in the United States, students' and their families' lives are deeply affected by the collegiate experience and how they adapt and perform in the first year—which often sets the stage for the balance of their stay. Because mathematics is considered key to access to and success in desirable fields of study and occupations (e.g., engineering, computer science), the transition into collegiate mathematics is often a special concern.

The research reported in this paper reviews the mathematical experiences of 8 students who moved out of a high school mathematics experience that was structured by current reforms in curriculum and teaching and into a more traditional collegiate mathematics experience. The university site for this work was Michigan State University in East Lansing, Michigan, a very large, land-grant, research university with an equally large and research oriented Department of Mathematics. All 8 were freshmen in Fall 1999 and since then have taken multiple semesters of collegiate mathematics. Our analysis describes and documents the challenges that many of these students have faced at MSU, mainly in Calculus I, and how they dealt with these challenges.

Our paper unfolds as follows**.** First we place our work at the MSU site within the work of the Mathematical Transitions Project as a whole, before describing MSU's Mathematics Department's course offerings for first and second year students. Then we present our research questions, say a bit about the character of the data we have collected, and describe our efforts to recruit student volunteers and the group of students who agreed to work with us. Then we present the results of our analyses: (1) how these students have performed in their mathematics courses, (2) what sorts of differences they reported to contrast their high school with their collegiate mathematics experience, (3) whether we feel they have experienced a mathematical transition or not. As we will explain below (see also Smith & Berk, 2001 for a more detailed discussion), we conceptualize mathematical transitions as involving changes students' disposition toward mathematics and in their approach to learning mathematics, as well as on changes in performance and reported differences.

The Place of the MSU Site Within the Larger Project

The Mathematical Transitions Project is currently examining students' experiences in mathematics as they move between fundamentally different curricula and teaching at two points in time—entry into high school and into college. At the MSU we study students' experiences as they move from experiences shaped by high school "reform" curricula to a more traditional program of collegiate mathematics. We have sought out and analyzed four main aspects of their experience: (1) how different students see their high school and college mathematical experience; (2) whether their <u>disposition</u> towards mathematics changes as they move into college; (3) whether they alter their <u>approach to learning</u> mathematics; and of course (4) whether their <u>performance</u> changes significantly. In this paper we will explore whether (or not) 8 students experienced a "mathematical transition" as they moved into collegiate mathematics. In cases where we assert that a "transition" happened, we will examine the nature of this transition.

Our site, Michigan State University, has been the context for exploring the effect of shifts from Standards-based high school curricula to more traditional collegiate curricula. In particular, our MSU participants worked with one of the two Standards-based curricula in high school: the Core-Plus Mathematics Project (CPMP) (or "Contemporary Mathematics in Context") or a curriculum developed "in-house" by mathematics teachers working in collaboration at a nearby high school that we will call Hogan High. They restructured the mathematics curriculum, from Algebra I forward, around the study of functions in realistic quantitative situations, exploration of these functions in tables, graphs, and equations, extensive sense-making discussions, and alternative assessments (performance-based exams and long-term projects). Team-teaching was also an important vehicle for reconceptualizing learning, curriculum, and teaching at Hogan.

<u>i. The Core-Plus Mathematics Project (CPMP)</u>

Core-Plus is an integrated mathematical sciences curriculum for high schools, consisting of a sequence of three year-long "core" courses for all students and a fourth year course continuing the preparation of students for college mathematics. Each year of the curriculum features four interwoven strands: algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. The content is built around solving problems embedded in real-life situations, with a focus on modeling and exploration with graphing calculators. The teaching approach emphasizes group-work and learning from peers. For more descriptive information on CPMP, see the curriculum's home page (www.wmich.edu/cpmp). The U. S. Department of Education's evaluation of CPMP and other mathematics curricula are also available on-line (www.enc.org/professional/federal/resources/exemplary/promsing/documents).

We mention here (and return to the issue below) that CPMP was published first as an integrated 3-year program designed for all students. The fourth year course, designed as an introduction to collegiate mathematics for the college-bound student, has just recently been published. So it is likely that the students who used these materials in high school from Fall 1995 to Spring 1999 did not work with the 4th year materials.

ii. The Hogan High Reform Curriculum

What we call the "Hogan High reform mathematics curriculum" was the result of an "in-house" effort to reorient curriculum and teaching in the spirit of the 1989 NCTM Standards. To deliver a curriculum that was more powerful and meaningful for students, a group of Hogan teachers recast the content of the traditional high school course sequence, while retaining their names: Algebra I, Geometry, Algebra II, Pre-Calculus, and Calculus.[1] The concept of function became the principal object of study. Quantities, co-variation, multiple representations, and linear functions were the focus in Algebra I. Successive courses (other than Geometry) progressively deepened students' knowledge of functions, their properties, and their uses.

The focus throughout was that functions were an essential tool for understanding mathematical relationships in the everyday world. Many of the problems were situated in realistic contexts where quantities change and co-vary, and much of the mathematical work could be described as "modeling" the relationships in those contexts. The curriculum was structured by a shared database of problems designed, edited, and annotated by the teachers. Teachers selected problems from the database to address the mathematical issues they were currently working on in a particular class. Lessons could also explore and build on students' ideas and conjectures from class discussions or from their projects and explorations. Textbooks were available but were not generally the central "carrier" of the curriculum. Each course was structured by a set of topics and issues, but teachers were free to draw and adapt problems from the central database as they saw them fit with the evolving work of their students. Algebraic manipulation was not neglected, but it was not the central focus of the work.

Students' work centered on discovering and exploring mathematical ideas and alternative models and solutions to problems. They were encouraged to work in groups most of the

[1] Not all teachers at Hogan have participated in this reform effort, though the majority have. Teachers who did not participate taught more traditional courses (curriculum and teaching) using University of Chicago textbooks (UCSMP). As a result, Hogan High students differed in how much their mathematical experience was influenced by this "in-house" reform.

time, in and out of class. In whole-class discussions students presented conjectures, tested their conjectures, and developed appropriate models for problems and solution methods. Graphing calculators were essential tools in these activities and were available for students in all classes. Assessment was based on a wide variety of activities: extended duration projects, class participation, quizzes and tests, and oral presentations around specific problems.

Overall, the Hogan High reform curriculum shares many features with CPMP: a focus on mathematical modeling, function as a central idea in algebra, extensive use of group work, exploration with graphing calculators, and written and oral expression of mathematical reasoning. It differs in the larger roles played by teachers in making curricular decisions: In choosing how to use specific problems to address particular mathematical issues in their classes.

The Introductory Mathematics Program at MSU

To fulfill the university's mathematics requirement, all MSU students must take a minimum of one mathematics course at the100-level or above. As is the case at many universities, there are many mathematics courses and sections (at the 100-level and below) that review some portion of high school mathematics. As explain below, we elected not to recruit students from all of these courses but restricted our efforts to pre-calculus and calculus. For that reason, we use the balance of this section to describe MSU's pre-calculus and calculus courses, because those were the classes where our participants' experiences were primarily situated.

MSU offers three calculus programs that use quite different curricula and are organized in different course formats. The first is a sequence of four traditional semester courses (Calculus I-IV: Math 132-133-234-235) covering limits, differentiation, integration, vector analysis, vector and multivariate calculus, and differential equations. The textbooks for these classes were authored by Thomas and Finney (1996; 1998). The last three classes in the sequence are taught in a large lecture (at least 75-80 students), small recitation group (≤ 30 students) format. But the Department has committed itself to teaching Calculus I in all small sections (≤ 30 students). The vast majority of sections are taught by regular or visiting faculty or instructors holding (or close to holding) doctorates in mathematics. The courses in this sequence are required for engineering, natural science, computer science, and mathematics majors but are also taken by many students who are not majoring in these "technical" fields. Mathematics and engineering students must take the entire sequence. This sequence has far larger enrollments than the other two calculus programs.

The Department also offers a two semester course sequence primarily for life science, social science, and business majors ("Calculus with Applications I and II," Math 124 and 126). These courses focus on differentiation and integration from a more practical and situational perspective than Math 132/133. The textbook is the Harvard Consortium's "applied calculus" text (Hughes-Hallet, Gleason, et al., 1996). Enrollment in this sequence drops substantially in second semester because many majors require only one semester of calculus. Third, the Department offers a four semester "honors" sequence that enrolls a small number of talented students who mostly intend to major in mathematics, science, or computer science. None of our project participants took honors coursework in mathematics.[2]

Pre-Calculus (Math 116) is an intensive, 5 credit review of the properties and behavior of functions—linear, quadratic, polynomial, rational, and trigonometric, and other topics deemed necessary for success in Calculus I (Math 132). The textbook for the course was authored by Bittinger, Beecher, Ellenbogen, and Penna (1997) and emphasizes graphs and equations of functions. The course combines 3 lectures (with ≥ 100 students) and 2 recitation (≤ 30 students) meetings each week. The lectures are given by tenured faculty and one experienced instructor.[3] Alternately, students may choose to take Math 103 ("College Algebra") and Math 114 ("Trigonometry"), a two-semester series that covers essentially the same material as Math 116, but at a slower pace.

Course placement is governed by two main factors: the student's ACT (or SAT) Mathematics score or their score on the 28 item Placement Test that mainly assesses high school algebraic skills. If incoming freshmen have ACT scores of 28 (or better) or SAT scores of 640 (or better), they are not required take the Placement Test. For those students, course placement depends on advising judgments based on high school course work and performance. For others, Placement Test scores of 19 (or better) allow students to enroll in Calculus I (Math 132) and scores of 12 (or better) allow them to enroll in Pre-Calculus (Math 116). These placements are not mandatory; students may choose a class with a lower course number (and assumed background knowledge), but not a higher one.

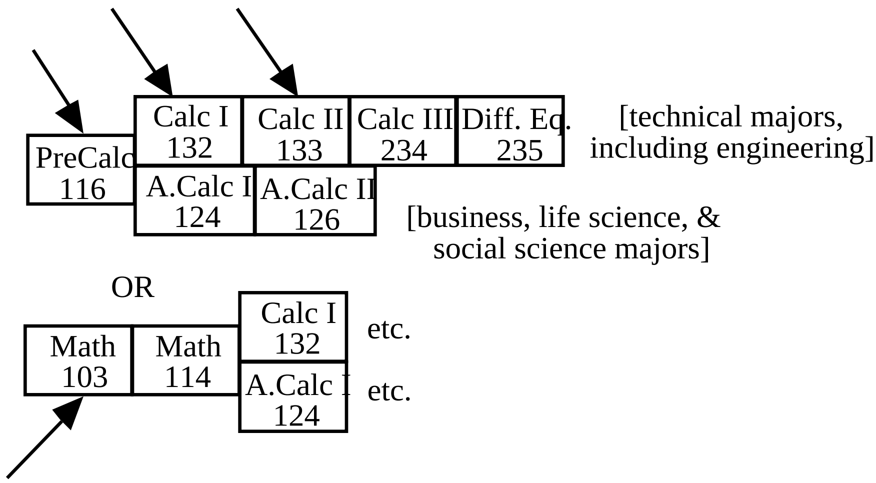
Because the Department recognizes that success in calculus is crucial to future enrollment and achievement in mathematics, it sees Pre-Calculus and Calculus I as critical sites for assisting and retaining students. As mentioned above, Calculus I is taught only in small sections by seasoned instructors to assist incoming students in making the transition to collegiate mathematics. The Department also reserves some small sections of Calculus II for freshman for the same reason. Some Pre-Calculus instructors attempt to enrich their lectures with use of graphing calculators and engaging mathematical situations. The Department also maintains an extensive Math Help Center with different rooms and tutors for large-enrollment courses. Additional resources are available on campus for students seeking assistance with their mathematics coursework.

The two large-enrollment calculus sequences are summarized below in Figure 1, with common entry points indicated by arrows.

Figure 1
Overview of the Two Main Calculus Sequences

[2] One student, TV, enrolled in the second semester of this sequence but dropped the course shortly thereafter due to the demands of her computer science major.
[3] This instructor holds a Ph. D. in mathematics, has taught the course frequently, but is not a regular faculty member.

(with common entry points)



Research Questions & Data Collection

Our conceptualization and exploration of students' mathematical experiences, here at MSU and more generally in this project, have led us pose, revise, and restate our research questions in a series of cycles (see Smith and Berk, 2001 for details). In our analyses of data from the MSU site for the 2001 AERA conference, we have focused our analysis on three main questions:

- What do students notice as different as they move from reform-based high school curricula and teaching into a more traditional program of collegiate mathematics?
- Who experiences mathematical transitions and why?
- What are some of the factors that influence their occurrence?

Our analyses below address the experiences of students who entered MSU as freshmen in Fall 1999. At this writing, they are completing their 4th semester of college. Our analyses draw on data we have collected from them over 3 full semesters of participation. During this time we interviewed them (2 or 3 times each semester), collected their grades and work from their mathematics courses, learned about their educational and career goals, assessed their beliefs about mathematics and about learning mathematics, and tried to track their weekly experiences in their mathematics courses in journals.

The Sample

According to our overall design (see Smith & Berk, 2001), we initially sought a sample of 25 students who fit our target "reform profile" in high school. That is, they were either graduates of Hogan High School and took classes designed around the "in-house" reform in their 11th and 12th grade years or were Core-Plus graduates from high schools with solid implementations of that curriculum. We began our recruitment efforts in the Fall of 1999 but quickly experienced difficulties. It turned out that simply attending a high school that used the Core-Plus curriculum did not guarantee a mathematical experience shaped by that curriculum. Most Core high schools, it turned out, maintained two different mathematics programs, Core-Plus and a more traditional program. Also, we found a relatively small number of Core graduates who had placed into pre-calculus and calculus. As a result, we ended up with fewer students (n = 9) who fit our reform profile, and we also attracted some students whose high school mathematical experiences were quite traditional. Rather than awkwardly rescind our invitation to participate, we elected to work with this latter group for one semester to get a sense of their experiences as they moved into collegiate mathematics. We refer to these students as “contrast students” below because we plan to use their experiences (particularly the differences they report) as contrasts for our Core-Plus students from the same high schools.

In Year 2 of the project (Fall 2000), we recruited again to increase our sample and attracted 11 more volunteers. But recruitment was again problematic, and only 7 of those 11 fit our reform profile. The 9 students recruited in Fall 1999 constitute Cohort I, and the new 7 students recruited in Fall 2000 make up Cohort II. We describe each below, but our analysis in this paper represents only the experiences of Cohort I. Cohort II students are currently completing their second semester of college, and hence at most their second collegiate mathematics course.

<u>i. Recruitment</u>

Both cohorts of participants were recruited primarily by e-mail solicitation.[4] The university administration provided the project with a listing of students by high school and mathematics course placement. Students’ e-mail addresses were located, and invitations were sent to all students who fit our profile (high school and Math 116, 132, or 133). The e-mail invitation described the research goal as one of understanding how college students coped with the movement from high school to college mathematics. The issue of high school curricular background (reform vs. traditional) was <u>not</u> mentioned because we did not want to influence how students would report on their high school experiences.

We chose not to recruit in other 100-level mathematics courses, because we anticipated our project staff becoming spread too thinly across too many courses and sections. Indeed, it proved difficult enough with so many different sections of Calculus I and II. Also, we wanted to see how students new to the college learning environment worked to learn new mathematics—not only to review content they had already seen in high school. At it was, much of Pre-Calculus review content students had studied in high school. (In Cohort II, this was also true for some students in Calculus I.)

The Project Director met with all students who responded positively to the e-mail solicitation in a series of small group meetings. He reviewed the goals and methods of the project and answered students’ questions. Their questions were few in number but when they came they typically concerned how knowledge gained in the project would be reported back to students and teachers. (We found these good questions!) All students who expressed interest also consented to participate. Each was paid $250/semester (as a credit to their tuition bill) and were re-recruited each semester. In most all cases, we were

[4] Students from Hogan High School in Cohort II were recruited via a letter mailed home from school with the same content as the e-mail solicitations used with other potential participants.

successful retaining students' participation semester to semester. Students who did not enroll in mathematics the next semester were offered $60/semester for a reduced level of participation.

ii. Cohort I (starting in Fall 1999)

All together we recruited 16 volunteers in Fall 1999. These students had either graduated from one of 6 different high schools using the CPMP or from Hogan High and placed into either Pre-Calculus, Calculus I, or Calculus II in their first semester at MSU. When we initially interviewed 8 of these 16, we found that 7 had taken all traditional courses in high school and one had taken a mixed program of some CPMP and some traditional courses. Since these students did not adequately conform to our desired reform profile, we chose to gather data from them in Year 1 but have not analyzed that data very intensively—concentrating instead of those remaining 8 who did fit the profile. Since our numbers were quite small, we also "lowered the bar" a bit and recruited freshmen at mid-year who had placed into College Algebra (Math 103) and Trigonometry (Math 114) from Hogan or one of the CPMP high schools. This effort added one additional student, NG. These 9 students are described below in Table 1.

Table 1
Overview of Cohort I
Fall 1999 Freshmen Graduates of Core-Plus and Hogan High

Student	HS Program	Gender	Major	F99 Math	Sp00 Math	F00 Math	Sp01 Math
JM	Core-Plus HS1	M	Mech. Eng.	Calc I	Calc II	Calc III	Calc IV
JB	Core-Plus HS2	F	Pre-Vet→ Pre-Med	PreCalc	Calc I → App. Calc I	App. Calc II	None
MD	Core-Plus HS2	M	No Preference → Business	Calc I	No Math	No Math	Business Statistics
NG	Core-Plus HS2	M	Eng.→ Public Res. Manage.	College Algebra	Trig	Calc I	None
JC	Core-Plus HS3	M	Pre-Med.	PreCalc	Calc I	Left MSU temporarily	Left MSU temporarily
NL	Core-Plus HS3	M	Engineering→ Business	Calc I	Calc I	None	None
TB	Core-Plus HS4	F	Elementary Education	PreCalc	Calc I (dropped)	Elem. Math for Teachers	Elem. Math for Teachers
PB	Hogan	M	Engineering → Business	PreCalc	Calc I	Calc II	None
NW	Hogan	M	Pre-Vet → Pre-Med	PreCalc	No Math	No Math	No Math

As shown in Table 1, Cohort I was mostly male, with only two female students. They came with interests in various fields, though engineering and medicine were common. Most changed their thinking about intended major in their first year. Seven students from 4 different high schools worked with the CPMP curriculum, and 2 with the "in house" reform curriculum at Hogan High. All students who used CPMP materials completed at least 3 full years of that curriculum. At High School 3 (NL and JC) CPMP was the only mathematics program. The other 3 high schools maintained two mathematics programs (a traditional program and CPMP), where most advanced mathematics students typically took traditional courses. This pattern did not hold in our sample, however, as both JB and MD at High School 2 were considered "gifted" and chose the CPMP program, where most of their classmates did not. PB and NW, the two students from Hogan High, had a solid introduction to reform mathematics in their last two courses (Pre-Calculus in the 11th grade and AP Calculus in the 12th grade). The curriculum and the teaching in these courses had all the reform elements listed above (see the prior description of the functions-based mathematics program at Hogan High).

Of these 9, all but NW took mathematics for at least their first two semesters at MSU. All 8 (NW excluded) had some experience with Calculus I (Math 132). The experience of both females students in Calculus (JB and TB) ended before the end of the semester, though for different reasons. JB attended the class for two days before switching into Applied Calculus (Math 124). TB struggled with the content and the instructor in Calculus I, switched to a different section (and so to a different instructor), and then dropped the class in the 8th week of the semester on the advice of her second instructor. She continued to take the mathematics required for her Education major but did not return to calculus.

In the balance of this paper, we report in detail on the experiences of the 8 students listed in Table 1 (excluding NW) over 3 semesters of mathematics coursework. But before we go on we first describe the students in Cohort II. Since we recognized the limitations introduced by our small Cohort I sample, we also recruited more students in Fall 2000.

iii. Cohort II (starting in Fall 2000)

In recruiting Cohort II we tried to use what we had learned in Year 1 about CPMP high schools with two mathematics programs to target students with the desired reform profile more effectively. But we were only somewhat effective in our efforts. Despite the best knowledge that we could obtain about high schools we thought were only using CPMP, we attracted graduates of those high schools who had taken traditional mathematics courses! Moreover, the graduates of CPMP high schools where we had contacts and could confirm students' backgrounds in advance did not respond to our e-mail solicitations, even with some phone follow-up to the e-mail solicitation.

All together, 12 students were recruited in Fall 2000, one of whom left MSU after one semester.[5] Of the remaining 11, four students experienced traditional mathematics coursework in high school, and one more (AA) had a quite mixed experience, taking 3 years of Core-Plus and two years of traditional mathematics. Unlike the one "mixed" background student in Cohort I, AA completed the first three years of CPMP, albeit in a quite atypical way. She shifted frequently between Core-Plus and traditional courses.[6] Though her high school experience was mixed, we list her below with the other Core and Hogan students because of her extensive, if discontinuous experience with CPMP. The 7

[5] As much as we have been able to ascertain, her departure to another university had nothing to do with mathematics.

[6] AA is the only student we have worked with or heard of who did this.

students in Cohort II who fit the CPMP or Hogan High profile are listed below in Table 2.

Table 2
Overview of Cohort II
Fall 2000 Freshmen Graduates of Core-Plus and Hogan High

Student	HS Program	Gender	Major	F00 Math	Sp01 Math
AA	Core-Plus HS5	F	Pre-Med.	Pre-Calc	Calc I
DN	Core-Plus HS6	M	Engineering	Pre-Calc	Calc I
KH	Core-Plus HS7	F	English or Education	Calc I	No Math
JH	Hogan	M	Engineering, Material Sci.	Calc II	Calc III
KG	Hogan	F	Hospitality Management	Calc I (dropped)	No Math
MD	Hogan	F	Engineering	Calc II	Calc III
TV	Hogan	F	Computer Science	Calc II	Calc III hon (dropped)

All 3 CPMP students graduated from high schools that were not represented in Cohort I, adding to the overall diversity in our site sample. The 4 Hogan High graduates all took the same 11th and 12th grade mathematics courses that we required of Hogan students in Cohort I, Pre-Calculus and AP Calculus. Fortunately, Cohort II dramatically increased our overall female participation, adding 5 to our original 2 female students in Cohort I. Cohort II also diversified the range of our students' academic interests, adding computer science and hospitality management. When Cohorts I and II were combined, our overall site sample became a more reasonable (though still small) 16 students—9 males and 7 females.

iv. Contrast students (from Fall 1999 and 2000)

In Table 3 below we provide similar information on students who did not fit our profile but participated for either two semesters (Cohort I) or one (Cohort II). We term these "contrast students" because their experiences will allow us to compare the differences between high school and college mathematics noted by our "reform" students with those noted by students from the same MSU courses but more traditional high school backgrounds. For some students in Table 3 below, their intended majors are unknown to us.

Table 3
"Contrast" Students in Cohorts I and II
Fall 1999 and Fall 2000 Freshmen from Traditional High School Programs

Student	Cohort	HS Program	Gender	Major	Sem 1 Math	Sem 2 Math
EC	I	Traditional HS1	F	Supply Chain Management	Pre-Calc	Calc I
DL	I	Traditional	F	**??**	Pre-Calc	Calc I

		HS1				
AC	I	Traditional HS1	F	**??**	Calc I	Calc II
PH	I	Traditional HS2	M	Environmental Science	Pre-Calc	App. Calc I
TrC	II	Traditional HS6	M	No Preference	Pre-Calc	No Math
LR	II	Traditional HS6	F	Pre-Med	Pre-Calc	Calc I
MS	II	Traditional HS6	F	Agircultural Management	Pre-Calc	No Math
PM	I	Traditional HS8	F	**??**	Pre-Calc	No Math
JF	I	Traditional HS8	F	**??**	Calc I	Calc II
JE	I	Traditional HS9	F	**??**	Calc I	No Math
TS	I	Mixed HS9	M	**??**	Calc I	**??**
TaC	II	Traditional HS10	F	Literature or Education	Pre-Calc	No Math

Results

In this section, we present our analyses of the mathematical experiences of our 8 Cohort I students who fit our "reform-based" high school profile. We structure our presentation around the four main factors identified in our model of mathematical transitions: (1) mathematics performance (i. e, course grades), (2) disposition toward mathematics, (3) approach to learning mathematics, and (4) perceived differences between past and current mathematics program—though with more attention to the (1) and (4) factors. With respect to the first three factors, we looked for evidence of change from high school to college.

Primarily because it is often taken as the only important indicator of success, in mathematics and academic endeavors more generally, we begin with our analysis of the change in performance in mathematics coursework. That analysis shows that Calculus I (Math 132) was a challenging experience for 6 of our 8 students—though their reactions to that challenge were quite different. Next, we present and discuss how our students described the differences they saw between their high school and collegiate mathematics courses (curriculum and teaching). Third, we present our characterization of their mathematical transitions; that is, we apply the "decision rule" that is described in detail in a companion paper (Smith & Berk, 2001). Briefly, that rule determines that a mathematical transition has occurred when the student has experienced significant change on any two of the four factors identified above. Fourth, we present three case studies of students (MD, JB, and PB) whose experiences illustrate the diversity of mathematical transitions as we have conceptualized them.

Mathematics achievement

Table 4 presents our students' mathematics performance relative to their overall academic achievement (as measured by their grade point average [GPA]) from 12^{th} grade through three semesters of collegiate coursework. The 12^{th} grade GPA values represent the students' entire academic performance in high school; the collegiate GPA values represent performance for that semester only. Our judgments of whether mathematics performance had changed significantly (column 7) relative to overall academic performance were based on the following formula:

$$|\Delta \text{ Math Grade} - \Delta \text{ GPA}| \ ?{\geq}? \ .5$$

If the difference between the differences was greater (in absolute value) than .5 for <u>any one</u> of 3 time periods (12^{th} grade to semester 1, semester 1 to semester 2, or semester 2 to semester 3), the student's mathematics performance was considered to have changed significantly. To clarify the nature of that change, we also list the semester in which the change took place, the magnitude, and the direction of the change. A single "up" arrow indicates a change of .5 to 1.0, a double "up" arrow indicates a change of 1.0 to 1.5, and triple "up" arrow indicates a change of 1.5 to 2.0. As you scan the rows of Table 4, you will see that change in performance depended in some cases primarily on changes in overall GPA and in others primarily on changes in students' grades in mathematics.

Table 4
Changes in Mathematics Achievement by Student
12^{th} Grade vs. the First Three Semesters of Collegiate Mathematics

Student	Categories	12^{th} grade	Semester 1	Semester 2	Semester 3	Change in Performance?	Focal Course
JM	Course	Core 4 & Intro to Calc	Calc I	Calc II	Calc III	NO	
	Math Grade[2]	2.5	3.0	3.0	3.0	Sem. 2: ↑	
	GPA[1]	3.33	3.46	2.85	3.0		
JB	Course	Core 4	PreCalc	Calc I→ App. Calc I	App. Calc II	NO	
	Math Grade	4.0	4.0	4.0	4.0		
	GPA	3.99	3.85	3.4	3.82		
MD	Course	Core 4 & Pre-Calculus	Calc I	No Math	No Math	NO	
	Math Grade	3.9	3.5	No Math	No Math		
	GPA	3.88	3.68	3.91	4.0		
NG	Course	Core 4	103	114	Calc I	YES	
	Math Grade	4.0	3.0	3.0	Very low[2]	Sem. 2: ↓	
	GPA	3.6	3.0	3.5	?	Sem. 3: ↓↓ ?	Calc I
JC	Course	Core 4	PreCalc	Calc I	On leave	YES	
	Math Grade	**?**	2.5	1.5	N/A	Sem. 2: ↓↓	Calc I
	GPA	3.5	2.75	3.16	N/A		
NL[3]	Course	Core 4	Calc I	Calc I	No Math	YES	
	Math Grade	4.0	1.5	1.0	No Math	Sem. 1: ↓↓↓	Calc I
	GPA	3.5	3.0	2.7	?		
TB	Course	Core 4 & AP Statistics	PreCalc	Calc I→ drop	Elem. Ed. Math[4]	YES Sem. 1: ↓	
	Math Grade	3.6	2.5	N/A	4.0	Sem. 2:	Calc I
	GPA	3.5	3.1	3.59	3.38	dropped Calc I	

PB	Course	AP Calculus	PreCalc	Calc I	Calc II	YES	Calc II
	Math Grade	2.7	2.0	3.5	1.5	Sem. 2: ↑↑	
	GPA	3.55	2.66	2.82	2.67	Sem 3: ↓↓↓	

Notes:
(1) High school GPAs are cumulative; college GPAs represent one semester's work.
(2) NG did poorly in Calculus I in Fall 2000 but we are, at present, unsure if his grade was 1.5 or 1.0. He is currently involved in a study abroad program.
(3) 12^{th} grade math grades are the average math grade for all courses taken in the student's 12^{th} grade year, except for NL took who took his last high school math course in 11th grade.
(4) TB took Math 201 & 202, math content courses for Elementary Education majors, during Summer and Fall of 2000, respectively.

As the GPA entries in Table 4 show, all of our 8 participants were generally good students, in high school and in college. For the most part, their college grade point averages were 3.0 or better; only PB and NL substantially differed from that pattern. Three students, JM, JB, and MD, did not experience a change in their mathematics performance from high school. MD struggled to achieve his 3.5 in Calculus I, and JB looked over Calculus I for two days and then quickly shifted into Applied Calculus I. In contrast to most college students, JM's and JB's grades did not drop when they came to college, in mathematics or more generally. Two students (JM and MD) took a traditional mathematics course in high school at the end of the CPMP sequence, both of which were designed as a transition to collegiate mathematics, especially calculus. In JM's case, this course appeared to be pivotal in this success in the calculus sequence at MSU.

We note that we have violated our own rule in the case of JM. His mathematics grades were identical in semesters 1 and 2, but his GPA dropped .51 points over that period. According to our formula, that drop in GPA qualified him for a change in performance, but only because his grade change in mathematics was measured against his overall grade change. Because the numerical value of this change barely surpassed our threshold (±.50) and because his mathematics grade did not change at all, we decided to override our formula in this case. More generally, instances like this one are leading us to reassess our measurement scheme for assessing the change in students' mathematics performance.

The performance of the other 5 of the 8 students did change, and in all but one case the change was downward. JC, NL, TB, and PB all struggled in semester 1. JC's, TB's, and PB's performances in Pre-Calculus (Math 116) were all substantially lower than their 12^{th} grade performance, and NL did much more poorly in Calculus I (Math 132) than he did in 11^{th} grade. NG's drop in performance came later, in Calculus I in semester 3. He performed well in the College Algebra and Trigonometry but received a poor grade in Calculus I. Only PB's pattern included both a drop and a rise in performance (an issue we also discuss in more detail below). Another pattern is apparent from Table 4: All but two students (JM and PB, both engineering students) struggled in Calculus I. Three did poorly (NG, JC, and NL [twice]); one dropped the course midway into the semester (TB); one felt so lost she shifted into the other applied calculus sequence (JB); and the other took very serious action and altered his early failing grade (MD). For students in Cohort I, Calculus I was often a difficult mathematical experience.

Differences Reported
High School vs. College Mathematics Courses

As at other sites (see Jansen & Herbel-Eisenmann, 2001; Smith & Burdell, 2001; Star, 2001), we found the analysis of differences to be a challenging, yet important task. Our 8 students' ways of comparing their high school mathematics and their college mathematics programs were quite diverse. Also, there were times when it was unclear how (if at all) we could use one of our scoring categories to capture the issue our student was identifying (see Appendix 1 for the current differences coding scheme). Though our analysis (and our coding scheme) will continue to evolve over the course of the project, we believe that our current analysis, given below, expresses our 8 students' views faithfully and relatively completely.

General developmental changes

As we have argued elsewhere (Smith & Berk, 2001), we wanted to discriminate, in our data collection and analysis, between those differences reported by students that were general and not directly mathematical from those that were. We characterize the former as "general developmental changes." They concern the impact of changes in the overall academic (and for college students, social) environment. At MSU, these general differences frequently concerned the relative level of parent (and other adult) direction vs. personal autonomy in how students chose their activities and allocated their time.

One student, JM, was quite explicit that the change in how he experienced his collegiate mathematics classes, especially in Semester 1, was influenced substantially by more general changes in his life and environment. He saw a dramatic change in how the expectations of others influenced how he structured his days. In high school, he felt that his parents and extra-curricular activities more or less dictated when he would go about his school work, generally and in mathematics. In college, he felt much freer about when, where, and how long he worked on academic tasks. He also saw that this change in his environment influenced his study and work activities. Where he resisted the directives of his parents in high school and tended to procrastinate, he welcomed the freedom in college to make his own choices. And as Table 4 indicates, he responded to his new-found freedom well—better than many college students.

By contrast, NL felt similar constraints in high school, primarily from his parents. In college, particularly in semester 2, he exercised his new freedom extensively, taking many trips away from campus and missing meetings of his classes in the process. Though his overall GPA did not suffer as much, his performance in mathematics dropped precipitously and remained poor. He could have elected to use his "free" time to work harder and try new learning strategies in Calculus I but he chose not to. But more to the point here, like JM, he was relieved to be making his own decisions about when, where, and how he worked on his course assignments. That he made poor choices from a performance and achievement perspective should no obscure this point.

<u>Mathematical differences</u>

We present the main mathematical differences reported by our 8 students in Table 5 in three clusters. The first cluster of 5 differences referred to issue of mathematical content, and in three cases involved perceived changes in the characteristics of typical problems. Not surprisingly perhaps, the students noticed and reported differences that clustered around a comparison of problems based in realistic situations that aimed at understanding (in the students' terms, "knowing why") vs. symbolic problems that emphasized the recall and application of algebraic formulae. The first two differences were entirely situated in Calculus I, where the students found the content, including the homework problems, more difficult than they had experienced before.

Table 5
Frequently Reported Mathematical Differences

Difference	Who mentioned
The college content is more difficult (calculus only).	JM, JB, MD, NG, JC
College homework is much more challenging than high school.	JB, MD
Typical problems are different in college. There are many fewer problems based in situations.	PB, JM, JB, NL, TB
High school problems focused more on concepts, understanding, and knowing why something works.	NG, JM, TB
College problems focus more on algebraic manipulations and memorization of formulas.	NG, TB, NL, JB
There is much less group work in college classes.	JM, JB, PB, MD, NG, JC, TB, NL
Much more class time in college is devoted to lecture.	MD, PB, JC
Graphing calculators play a reduced role in college classes.	JB, MD, NG, NL
The pace of instruction was substantially faster in college.	JM, JB, MD, NG, JC
College instructors do not invite and seem less interested in student participation.	NL. JC, JM
Close relationships with teachers are harder to establish in college.	JM, NG, PB, NL
Spoken or written communication with teachers was often difficult.	JM, JB, PB

The second cluster of 4 differences concerned various aspects of the organization of lessons and learning in the classroom. While graphing calculators use was encouraged in some MSU mathematics classes, their use was not allowed in exams so students had good reasons to restrict their use. Likewise, they saw that the increased time devoted to lecture resulted in decreased time to work in groups. Only JB reported that she disliked the group work in high school that the others liked. And, not surprisingly, students reported a quicker pace of instruction in college, though some (like JM) excused their professors from responsibility because they knew that the Department had mandated covering a fixed amount of content in each section.

The third cluster of differences referred directly to the stance and work of college instructors. Some students did not feel that their instructors were very approachable and during class were not deeply interested in students' questions. This difference is consistent with their perceptions about the pace of instruction and the amount of lecture presentation. And finally, some students did report difficulty seeing what their instructors had written on the board or understanding what they said in class.

Overall, we see that the entries in Table 5 fall into two main categories: (1) differences that reflect important design features of reform curricula (specifically, CPMP and the Hogan High curricuum), assessment, and methods of organizing learning, and (2) differences that are characteristic of college teaching, both generally and in mathematics.

Mathematical transitions
Who Experienced One and Why?

One of our project tasks has been to develop a working definition of "mathematical transition" in order to apply that definition to the data we have collected from our project students. According to our working definition, students have undergone a "mathematical transition" if our analysis indicates <u>significant change on two or more dimensions</u> of their mathematical experience:

- a significant change in their performance
- a significant change in their disposition towards mathematics
- a significant change in their approach to learning mathematics
- significant differences reported between their present and prior mathematics program

In the two previous sections we showed how we decided on significant changes in performance and reported significant differences. To assess change in <u>disposition towards mathematics</u> we used three indicators: (1) a change academic goals based on their mathematical experiences, (2) a change in their attitudes towards the subject (primarily, " like/dislike"), and (3) a change in beliefs about mathematics and about learning mathematics measured by the Conceptions of Mathematics Inventory (CMI). The CMI survey was developed by Douglas Grouws and colleagues at the University of Iowa (Grouws, 1994). Because students' academic goals could change for many reasons (not all related to their disposition toward mathematics) we looked especially for changes in attitude and/or belief. To assess changes in students' <u>approach to learning mathematics</u>, we looked for evidence that they developed new learning strategies, laid aside old strategies that did not seem to work, or used existing strategies differently in their efforts to learn mathematics. We only counted actions that students undertook on their own, not those mandated by their teachers.

Table 6 presents our summary judgments for our 8 students in each of the 4 domains of their mathematical experience and our overall judgment about transitions (yes/no). When we feel we have evidence that difference noticed, changes in learning approach, and changes in dispositions were centered in the students' experience of Calculus I (Math 132) we have noted that in the table.

Table 6
Mathematical Transitions

Student	Δ Math Performance	Δ Math Disposition	Δ Learning Approach	Significant Differences?	Mathematical Transition?
JM	No	No	Yes	Yes?	Yes?
			In Calculus I		
JB	No	No	No	Yes	No
				In Calculus I	
MD	No	No	Yes	Yes	Yes
			In Calculus I	In Calculus I	
NG	Yes	Yes	Yes	Yes	Yes
		In Calculus I		In Calculus I	
JC	Yes	Yes?	No	Yes	Yes
NL	Yes	Yes?	No	Yes	Yes
				In Calculus I	
TB	Yes	No	Yes	Yes	Yes
PB	Yes	Yes?	Yes	Yes	Yes
			In Calculus I		

Though our analysis remains tentative in places (witness the question marks in some cells), our current position is that all but one or two students experienced a mathematical transition. Only in JB's case are we comfortable with our "No" judgment overall, though her "no change" scores in performance, disposition, and learning approach may well have resulted from her shifting out of Calculus I and into Applied Calculus I. JM represents a more tentative case. He reported a significant number of differences between his Core-Plus high school classes and his calculus classes at MSU, and his approach to learning changed at MSU. But in many ways he was ready and waiting for these differences and changes and took them very much in stride. His traditional "Introduction to Calculus" class as a senior high school (see Table 4) played a particularly central and direct role in preparing him for mathematics at MSU. The third student whose performance was not effected, MD, was, by contrast, a very clear case of a transition. He was deeply effected by Calculus I, saw it as very different than his high school Core classes, and had to make major adjustments in his patterns of work and learning to succeed.

This pattern of diversity continued among the 5 students who performance did drop in mathematics, relative to their overall academic performance. First, all of these students' mathematics achievement dropped from their high school levels, relative to the their GPAs. Beyond that, there is no single pattern to be found. JC and NL both struggled to achieve and noticed significant differences, but did not change their approach to learning in any significant way. In NL's case, this is particularly remarkable; he did very little different even in his second attempt in Calculus I. Both JC and NL's disposition toward mathematics became more negative, but in ways tied to the specific course they struggled with, Calculus I, and even there not so strongly. NG and PB both experienced change in all four dimensions, but their cases were yet still quite different. PB’s performance fluctuated (down, up, and down again), and the greatest changes he made in his approach to learning mathematics came in the semester when he was most successsful (Spring 2000 in Calculus I!). NG, by contrast, was successful initially in a year’s worth of review (College Algebra and Trig) but performed poorly in Calculus I despite trying to adjust his learning methods to the very different content of the course.

We can also consider the nature of these mathematical experiences by looking down the columns of Table 6. All students, except possibly JM, noticed and reported significant differences between high school and college. Half of them gave clear indication of changing the way they went about learning in reaction to these differences. But only in a few cases did we find clear evidence that the students’ disposition changed. Indeed, we have found it difficult to assess their dispositions toward the subject-matter (more or less positive) in a way that does not reflect merely their performance (better or worse) or the characteristics of their particular current courses.

To summarize, two kinds of results stand out from Table 6: (1) Students were assessed as having experienced a mathematical transition for very different reasons, and (2) the dynamics of those transitions played out differently over time. For some, like MD and NL, challenges and difficulties were immediately felt; for others, like NG and PB, the challenge did not occur at the entry to college.

A peak into the particulars:
Lessons from individual students

In this section, we try to place some experiential “flesh” on the “bones” of our analysis. We present three short case studies of students whose mathematical experiences (and actions) were quite different. One (MD) felt an immediate and strong impact of a very different kind of mathematics in college. He reacted, with help, to this jarring experience and ultimately had a “successful” transition. Another (JB) did not, we think, experience a mathematical transition, primarily because she acted to reduce the difference between her high school and collegiate experiences. A third (PB) experienced a mathematical transition but one that was factored over multiple semesters. His case reminds us that we must examine events over an extended period of time to assess students’ reactions to reform to traditional curricular shifts. Not all effects are immediate, and the timing of challenges depends on a host of factors.

<u>MD: Challenge, adjustment, and success in Calculus I</u>

MD is an instructive case because he experienced the same sense of dislocation and challenge in Calculus I reported by other participants (NG, NL, JC, and TB), but he made a strong and very serious effort to rise to the challenge of a very different mathematics. He was eventually successful, earning a 3.5.

A graduate of the same high school as JB and NG, MD was in the "gifted" mathematics track and took Algebra I in 8th grade. On the advice of his parents who were positively impressed with the Core-Plus curriculum, he shifted into the Core "track" in 9th grade and took all three years of it (Core 1-3).[7] In the 11th grade (second semester), he also took a hastily prepared Pre-Calculus course that was offered in response to pressure from parents, who were concerned about a lack of match between Core and collegiate mathematics.[8] In 12th grade he took AP Statistics and got a 3 on the AP test. By the time we met MD in college, he described himself as "poor" student in high school—someone who did enough to get by but did not really engage deeply enough in what he studied to master it. His self-attribution was general, but it applied to his work in mathematics. He reported that he was anxious for the pace of instruction to speed up in his Core classes; he sensed he learned more quickly than his peers.

As a first semester freshman (Fall 1999), he placed into Calculus I (Math 132) on the basis of his ACT score. Almost immediately, he felt as if he was a fish out of water, and he struggled. He reported that he would stop listening and thinking at times during lecture and when he "came to"," he had lost the thread. He felt the content was profoundly different, more difficult than high school, and the pace of instruction was much faster. After receiving a very low grade on the first exam (< 50% correct), he went to see his professor. Feeling he had been placed into the wrong mathematics course, he was ready to drop Calculus I. His professor took his situation seriously, asks a few questions about his high school background, and eventually convinced him that he <u>was</u> in the right course. She worked with him to fill in some content where he seemed to have weaknesses (specifically, some aspects of trigonometry) and urged him to develop some new learning and study skills. MD responded well, used her help extensively (i.e., meeting with her twice a week), and became very disciplined about when, where, and for how long he worked on his assignments. Specifically, he went to the Math Help Center every day after class and completed his homework before he allowed himself to go home. If he experienced difficulties on any problems, help was readily available. MD did not take mathematics in either of the two following semesters (Spring and Fall 2000), but in Spring 2001 he enrolled in Statistics 315, a required course for business majors, and at this writing is doing well there.

MD was unique in Cohort I in his serious and sustained efforts to change how he went about learning mathematics (that is, the change in his learning approach). While others struggled with the content and teaching in Calculus I, no one in Cohort worked as hard or as effectively to adjust as MD. He is also unique in that he was only student in Cohort I who developed a working relationship with a regular faculty member. MD acted to meet the challenge, but the university also acted to encourage and hold him in Calculus I. In MD's eyes, the experience of "being lost" and rising to meet that challenge fundamentally changed his entire college life. It appears that his eventual success in a class he thought he should drop has given him the confidence to believe that he can

[7] His high school, HS2, continued to offer a traditional mathematics program, along with Core-Plus. During MD's years at HS2, only three years of Core were available. The 4th year curriculum was still in development.
[8] We take no position on the veracity of these claims; we simply retell the story as MD related it.

master any course at MSU—and his GPA bears out that belief thus far. As a footnote, he sees his statistics class as somewhere "in-between" Calculus I and his high school Core classes. Statistics 315 and Calculus I share some aspects of teaching and organization (both are lecture-based course with little interaction), but the latter stands out for him as the very different from his high school experience (and substantially different from Statistics 315). In particular, he sees the nature of the statistics content and the kind of problem posed much more like his Core-Plus experience than Calculus I.

JB: A two day transition: Choosing out of Calculus I

JB's case is remarkable because she was the only one in our 8 students whom we did not feel experienced a mathematical transition. On the basis of her strong high school performance, JB entered MSU on an academic scholarship. She applied and was admitted to the Honors College and continued her strong academic performance from high school. As Table 4 indicates, she was and continued to be an "A" student. Her exposure to traditional mathematics at MSU involved two courses: Pre-Calculus where she adapted easily and Calculus I where she did not.

Like MD, JB took all three available years of Core-Plus in high school, distributed over 4 years. More so than MD, JB was critical of the program, focusing her objections on the group work (where she felt she had to "teach" slower students) and the "psychological" aspects of some problems where questions and responses did not seem "mathematical" in character. On the other hand, she was proud of her performance and Core's contextual problems helped make mathematics meaningful. Based on her ACT score, she placed into Calculus I, but she decided to take Pre-Calculus instead because she wanted to reduce the demands that her mathematics course made on her time relative to her other courses. JB entered MSU in the Pre-Veterinary Medicine program.

She cruised through Pre-Calculus noting differences between it and her Core classes, mostly around typical problems, the absence of group work (which she appreciated!), and the lecture-discussion format. Much of the content was review for her (and most others in Cohort I who took it). But the situation changed abruptly in January 2000 when she enrolled in Calculus I. She stayed in the course for only two meetings, but its impact on her mathematical experience was strong and compelling.

She saw the course as dramatically different from both her high school Core classes and from Pre-Calculus at MSU. She focused on the symbolic and decontextualized nature of the content; the abstract and formal presentation of delta-epsilon proofs of limits (the purpose of which were not introduced or motivated); the impersonal and non-interactive nature of the teaching overall (the professor did little to connect to the class); and the work with the textbook that was required to "unpack" the content and understand and solve the assigned problems. JB struggled for two days to prepare for the second class meeting. In those two days, she concluded that (1) she could succeed in the course only by devoting a very large amount of time and effort and (2) how she worked to learn the content would have to change as well. For example, she was used to working independently and she was unsure that she could do that and succeed in Calculus. Also, she was not ready to focus on this class, given the impact of that choice on the rest of her coursework.

When she found that MSU offered an alternative calculus sequence and the Harvard Consortium textbook looked much closer to her Core high school experiences, she dropped Calculus I and enrolled in Applied Calculus I (Math 124). She quickly found that course and its successor (Math 126) more to her liking. The content fit perfectly with her Core experiences, the problems were the familiar situational type, the instructor was accessible and interactive, and the group work was not enforced—so JB could consult with other students only she needed to, which was rarely.

She cruised through both semesters of Applied Calculus, reporting few difficulties. The reduced demand allowed her to focus on her science courses, which were her priority. But interestingly, her assessment of aspects of her Core experience in high school changed as a result of the shift from Calculus I to Applied Calculus. By comparison to Calculus I, JB felt motivated to learn and enjoyed her work in Applied Calculus, where she saw no utility for learning the traditional symbolic approach. She sensed her deep understanding of the differentiation and integration in contexts of application and liked it. At the heart of this comparison were the “real-life” situations and problems that formed the heart of the Applied Calculus curriculum. She even came to a reassessment her work to “teach” her peers in small groups in high school. She realized that these efforts had helped her build the strong understanding that she brought to Applied Calculus.

We can only speculate on JB’s potential experiences if she had stayed in Calculus I. Her preoccupation with high grades governed her decisions about her coursework, even more than her orientation toward the content. If an alternative calculus track were not available, we expect that JB might well have eventually faced down the challenge, if she had successfully adapted her learning approach to the character of the Calculus I content. But in the process, her disposition and performance could have changed as well.

<u>PB: Transition Amid a Fluctuating Performance</u>

PB is the only student in our group of 8 students who graduated from Hogan High School, where a reform mathematics curriculum was developed “in-house.” (NW, the other student in Cohort I from Hogan, only took one semester of mathematics in his first two years.) PB’s case is interesting because his performance was unique in its fluctuation. He did poorly in Pre-Calculus relative to the amount of content that was review; he rebounded strongly in Calculus I, despite the demands of that course; but then he fell again in Calculus II—a set-back that caused to leave his intended engineering major for business.

PB’s achievement in high school mathematics was not particularly strong, and his reception of the reform elements of the Hogan curriculum was mixed. He also noted that group work was used extensively in high school but not in college.

> [Mr. C, high school calculus teacher] in high school tries to teach his class through discussion. The group teaches you instead of him…That style of teaching… I don’t really enjoy it too much. I like to have [a] lecture.

PB also noted that assessments, typical problems, and the general educational environment differed significantly from those in his high school experience.

> I remember in high school, we did a lot more thinking about real-life situations. We did a lot of [group] projects and…if you paid attention and worked on it, you'd get a good grade. Well, here you have a test, a quiz, a problem set. If you don't perform, well <u>there</u>. Then there's no hope for you. It's not like you can't ask questions, but it's harder...I enjoy a smaller class…I think [college is] really impersonal… "You're there to listen, so listen." But when you're in high school, …[the teachers are] gonna make you, they're gonna almost force you to learn it…I guess when you're in…a lecture hall that big, you just can't care about individuals.

PB did not do well in the large lecture Pre-Calculus course his first semester, earning a 2.0. He was consistently surprised at how poorly his test scores were relative to his expectations on leaving the exam room. He disliked his professor, who he regarded as indifferent to both his students and his teaching.

> [The professor] just went up there and…he has a really grumpy voice and kind of scribbled on the overhead for fifty minutes, just kind of went through… chewing his gum [PB snickers]. I mean, it's just little things like that that kind of clue you in…

He relied on old study strategies, studying with friends only just before for the tests, but these strategies were not as effective as they had been in high school.

PB redoubled his efforts second semester in Calculus I, after receiving a warning letter from the University about his grades. He attended an organized study group, involving four students and a MSU-employed group leader who knew the content well. The group met one hour per week to hold a mini-lecture and discuss homework questions. PB reported, "Now, I never leave without getting all of my questions answered." His grade of 3.5 in Calculus I reflected that strengthened commitment. But PB also felt supported by his new instructor. "[He] goes through and explains everything both in class and when he's grading. He writes all the comments right there on your paper." PB attributed his improved grade to several factors in addition to his new professor: "… I think a smaller class made it so much easier to focus on the teacher. Finally, I think that the 2.0 I received in 116 really woke me up. I realized, I can't coast through college the way I coasted through high school. That just won't cut it here."

Unfortunately, PB's success did not continue. His third semester (Fall 2000) Calculus II course was very difficult for him. He reported continuing all of his study habits from the previous semester, with the exception of the MSU-organized study group, although he was again enrolled in a large-lecture course. He rated his instructor's teaching as average. His grade dropped to a 1.5, after which he changed his intended major from engineering to business. (Entrance to the College of Engineering requires a GPA of more than 3.0.)

Throughout his collegiate experiences in mathematics, PB's enthusiasm for the subject seemed to wax and wane, depending on his success. For example, in Pre-Calculus he was discouraged because his grades were lower than he expected. He "prepared" for the tests, but did not see the desired results. The next semester, when he realized that he was doing well in Calculus I, PB declared, “If I leave engineering, math won’t be the reason.” It was never completely clear whether his affinity for the subject was based on anything other than his performance and his attitude toward his instructors.

Overall, PB preferred and prospered in a small lecture format with an approachable and engaged instructor. He reported differences in the typical problems, the character of assessments, and the presence or absence of group work. He found the college environment generally impersonal, and professors rarely reached out to students. He felt that it was more difficult to ask questions and establish a working relationship with them. Though he is ultimately responsible for his performance and learning, his record clearly indicates that issues of teaching and course organization, as well as his willingness to alter his work and study patterns, had an impact of his (fluctuating) performance.

Discussion and Conclusions

In this closing section we identify and discuss a number of issues raised by our work at MSU thus far and our analysis of the mathematical experiences of these 8 students. One is methodological in nature: the problems of assembling a “reform” student sample at this site. Most others directly concern our results. Finally, we stretch a bit to consider how one MSU course, Pre-Calculus (Math 116), might serve students like ours better. Overall, we want to caution readers that our analysis reported here is preliminary. We have more to do in analyzing the experiences of even these 8 students, and we have not worked much at all on experience of the Cohort II students. More generally, readers are cautioned against generalization from the experiences of this small group to the larger population of students who have experienced either reform curriculum.

Constructing of an appropriate sample has been a challenging task. When we began this work, we did not expect substantial difficulties in assembling a collection of 25 freshmen who fit our desired profile of high school mathematical experience. Our experience has been quite different, for two main reasons. First, by limiting ourselves to three MSU courses (Pre-Calculus, Calculus I, and Calculus II), we have restricted the pool of potential participants. In fact, high schools that provide the kinds of experiences we are looking for (mainly the CPMP curriculum) are small in number, as are their students who place into those classes. Second, we naively thought (in Year 1 at least) that “high schools using the CPMP curriculum” used only that mathematics curriculum. We have found that not to be the case. In fact, the rule (rather than the exception) has been that CPMP is usually offered alongside a program of traditional mathematics courses. Because we did not initially appreciate this fact, we recruited “traditional” students from “reform” high schools before we realized it. That we also recruited such students into Cohort II, even after we appreciated the problem shows that it is not a simple problem to resolve.[9]

[9] In fact, knowing who to recruit from which high school requires having faculty contacts at each high school, something we are only now developing.

The resulting reduction in the size of our MSU sample, even after the addition of Cohort II, is a significant concern, and we are considering, even in the last year of funding, seeking a third cohort of students. Such an effort, if it takes place, will be preceded by visits to high schools before students come to MSU to identify and meet potential participants who fit the profile we seek. We hope that personal contact with the project staff will help encourage more students to volunteer.

<u>Mathematical transitions are not only a matter of performance</u>. We know that many who participate in K-16 education (students, parents, K-12 teachers, college faculty, and administrators at all levels) may strongly associate the notion of a "transition" in mathematics from high school to college with performance in collegiate mathematics, typically in calculus. Our work shows directly, we think, that this view perspective is too narrow. As MD's case shows, performance is not all that matters, in mathematics coursework and more generally. Students may perform successfully and either be affected deeply by the experience (as MD was) or not (as JM was). In general, successful performance can stop analysis of and learning from students' experience precisely where it should start.

<u>Difficult mathematical transitions can be extremely beneficial for students</u>. Again, MD's case illustrates how much benefit can result from a serious challenge in the content or the pedagogy (or both) of collegiate mathematics, relative to expectations from high school. MD believes, and we concur, that his experience in Calculus I transformed him as a student, from the typical "go along, learn a little" high school student who achieves easily to someone who is hungry for knowledge and expects to master his coursework in college. Such a transformation might have happened without his experience in Calculus I, but as a matter of fact, it did happen directly as a result of that experience. The challenge gave MD the confidence to develop his interest and seek out other challenges at MSU.

<u>Does CPMP prepare students for Calculus I at MSU ?</u> Calculus I has clearly proven to be a serious bump in the road for our students coming from three years of Core-Plus mathematics. Of the 7 graduates of Core-Plus in Cohort I, all but one (JM) experienced a major struggle with this course, either in terms of performance or overall challenge to learn and achieve. MD met a special effort to meet this challenge, while JB looked the same challenge and turned away from it. The remaining 4—NG, NL, JC, and TB—all experienced poor performances, whether they stayed through the course or dropped it (TB). MD and TB both reported they felt the professor in Calculus I was speaking a "foreign language."

Faced with this result, one could conclude that the Core-Plus curriculum (where these students had prospered) had not served these students well relative to the expectations of traditional calculus at MSU. While we find some validity to this interpretation (particularly with respect to the "relative to the expectations of traditional calculus" clause), we also find it narrow and misleading in several ways. First, the first three years CPMP mathematics is much more diverse in its mathematics content than the traditional, algebra-laden sequence of high school college-prep courses (Algebra I, Geometry, Algebra II, Pre-Calculus). In particular, CPMP teaches much more statistics and probability and discrete mathematics than the traditional mathematics programs. These topics seem to prepare students quite well for collegiate mathematics courses other than calculus—witness MD's assessment of Statistics 315 as unchallenging after Core.

Second, it is not clear how many, if any of the 7 Core-Plus graduates in Cohort I experienced the fourth year of the CPMP curriculum, where the explicit focus is preparation for collegiate mathematics, especially calculus. (We suspect that our students Core 4 classes included primarily the second half of the 3rd year materials.) For MD and NL, it is quite unlikely that they were using even pilot 4th year materials in their 11th grade year (1997-1998), which was their final year with Core-Plus experience. To determine this issue for sure, we will have to determine, high school by high school, what the site-specific content of "Core 3" and "Core 4" was in 1997/98 and 1998/99. Assessing the differential impact of the 4th year of CPMP on MSU students enrolled in Calculus I may well require a different, more recent sample of students. That is another reason why we are considering recruiting a third cohort in Fall 2001.

Third, we question the assumption that the content, instruction, and pace of Calculus I is the next mathematical experience for MSU-bound students. Many students would be served much better, at least in an instrumental sense, by a collegiate course in statistics. Moreover, it is unclear what mathematical depth students who are successful in Calculus I are building; success in the symbolic manipulation of limits, derivatives, and integrals hardly means that that students have learned about the rate of change and accumulation of functions (Thompson, 1994). Fourth, it is not yet clear how the performance of these 7 students compares to the performance of the entire Calculus I student population. At present, we know at least that the Department considers success in Calculus I important enough to staff it in all small section with regular and visiting faculty, yet many students still drop out or achieve poorly.

Pre-Calculus has not served a bridge to Calculus I for reform students. The design intention of Math 116 is to prepare motivated high school graduates for Calculus I. Yet, for our students and apparently more generally, this goal has been elusive. We know from our students that much of the Pre-Calculus content was review, and as a result they neither learned much new content nor developed new expectations for study and mastery of mathematics. From what we know, this assessment parallels the Department's; they are dissatisfied by the relatively small number of students who go on to Calculus I and are successful. Here we think we have a plausible explanatory hypothesis grounded in data that large-sample analyses could not provide support for. Our participants (JB, JC, TB, and PB) all failed to report feeling challenged in this course—whether or not they were ultimately successful or not. It seems that the mostly familiar content allowed them to move along without demanding mastery. TB's commentary on Pre-Calculus is indicative of how students may have been slow to rise to the challenge of a deeper or different level of mastery,

> ….. in fact the ideas that are also brought up in the lecture are kinda a repeat and review of what the course was last year [in 12th grade]…Not even just last year, because when we talk about slope and we talk about functions, those were freshman year things so sometimes a lot of parts of

> this course are really basic…When you get the lecture notes, and you think that a lot of things are really basic and in fact, a lot of them are, but then half way through your problems, you can see that they are forcing you to take it a little bit further than what he's talked about in class. You have to be able to take those connections and make them work for you to do the work.

These students mostly noticed the structural changes (e.g., the large lecture and small recitation sections where they knew no one) and complained, if quietly. Little in the course, at the beginning of the semester, required JC, TB, or PB to "raise their game." And indeed, it may not be the institution's responsibility to do that; in college, students are responsible for organizing their own learning. But the contrast of reactions to Pre-Calculus and Calculus I is striking, from all of our participants who saw both, except for PB. If this is the case, the Department might serve the needs of students who have taken pre-calculus in high school better if it explicitly organized the course to acknowledge the content review and posed problems that asked more of students. In short, the course might become harder in a more meaningful way.

<u>Students notice the design features of reform curricula</u>. Both the graduates of the CPMP curriculum and of the teacher-designed reform curriculum at Hogan High noticed and reported differences between their high school and collegiate experiences that show reform curricula have an effect on students' experience of mathematics. The students were quite variable in the character of their stance toward those differences. Some preferred aspects of reform curricula and teaching, some did not. Some noted differences without feeling strongly about them, one way or the other (JM), others expressed differences with strong affect (JB). But overall, we now know that students notice differences between (1) situational, contextual problems and decontextualized, symbolic problems; (2) the perceived focus on conceptual understanding vs. symbolic manipulation; (3) the organization of classroom learning around small groups vs. individual study; and (4) the central vs. peripheral role of graphing calculators. Given the overall goals of the designers of these curricula, it appears they have been successful in reorienting how students see, experience, and learn mathematics.

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Appendix 1
Mathematical Differences Coding Scheme
Mathematical Transitions Project

Curricular Differences

i. Surface characteristics
• appearance and/or packaging (e.g., color; layout; number and size of volumes, etc.)

ii. Mathematical characteristics
• comprehension of textbook's terminology (easier, harder, etc.)
• comprehension of passages in the text (descriptions, explanations, etc.)
• typical problems (e.g., more "story" than "number" problems or vice-versa)
  • ease/difficulty of comprehension (understanding what the problem asks for)
  • understanding of content required (e.g., more or less, deeper vs. shallower)
  • perceived value or relevance to student
• direction provided for solving problems (e.g., explicit solution procedures for specific problem types (highlighted boxes) vs. general verbal descriptions vs. no guidance)
• representations of mathematical relationships, other than verbal (e.g., tables of values, graphs, equations, diagrams)
• content is different (at the "top level," e.g., geometry as opposed to algebra)
• content is presented in greater depth and detail (in contrast to "new" content)
• content is more diverse mathematically
• some topics or problems seem non-mathematical (that is, outside of the boundary of "math" as defined by the student)
• absence of expected topics (e.g., algebraic manipulation)
• short-term coherence/connection among topics (within chapter or unit)
• longer-term coherence/connection among chapters or units (over a semester or year)

Teachers/Teaching Differences

• teacher-student relationship (e.g., accessibility, trust, care, contact outside of class)
• basic communication problems (literal comprehension: e.g., student cannot hear, understand, and follow the teacher's speech)
• classroom management (e.g., difficulties in focusing, following, or participating because of different classroom norms)
• teacher's typical lesson
  • activities in typical lesson (elements include: checking homework, lecture, experiments or data collection, individual problem solving, small group problem solving, large group discussion)
  • sequence of activities
  • duration or importance of activities
  • organization, direction, or shaping of activity (e.g., nature of class discussions)
• teacher's expectations for student participation (what teachers expect students to say and do)
  • nature of teacher questions
  • nature of student questions invited by teacher
  • discussion of alternative solutions to problems

• nature of group work
- • frequency of use (continuum from “never” to “a lot”)
- • size of groups (pairs, trios, fours, or larger)
- • selection (how groups are set up, e.g., student- or teacher-chosen)
- • perceived value (both “negative” and “positive” value)
- *• instructional pace (when set by individual teacher, not the Department as a whole)*
- *• teacher’s fidelity to the curriculum (how closely the teacher “follows” the book)*

Differences due to Site Policies

• instructional pace, when set by Department decision
• class size

Note: if class size difference is mentioned, list also the effect(s) felt by the student.

• duration of class meetings
• duration of courses (e.g., semester vs. year-long)
• assessment (when the content of the assessment is determined by the Department coordinator or committee, not the individual teacher)
- • kind of assessment (in-class tests, take-home tests, group projects, performance assessments, etc.)
- • problems on assessment (e.g., like or unlike what was “covered” in class)
- • time to complete assessment
- • assistance provided during assessment
- • overall difficulty (as seen by the student)
- • “additional” assessments, e.g., “gateway” tests)

Differences due to Individual Changes/Development

• motivation to learn/achieve in school generally (presuming the student does not attribute motivation change to others, e.g., teachers)

Note: In both motivation categories, list source of motivation change if known, e.g., change in student’s goals.

• motivation to learn/achieve in mathematics specifically (presuming the student does not attribute motivation change to others, e.g., teachers)
• ability to organize time and effort
• resilience to academic challenges and difficulties

Curriculum x Teachers/Teaching

• nature of expected solutions to typical problems
- • amount of verbal explanation
- • detail in “showing work” (numerical and symbolic steps to the solution)

• assessment (when individual teachers have substantial discretion to design their own)
- • kind of assessment (in-class tests, take-home tests, group projects, performance assessments, etc.)
- • problems on assessment (e.g., like or unlike what was “covered” in class)
- • time to complete assessment
- • assistance provided during assessment
- • overall difficulty (as seen by the student)

- typical homework assignments
    - number of problems
    - difficulty of problems

Site Policy x Teachers/Teaching

- changes in what students are expected to do on their own

Other

- composition of the class (who is there in the class with the student)

Note: we are scoring changes in how graphing calculators are used in class (e.g., nature of use, frequency of use) but we have not placed this category into our scheme.

Viewpoint: No, the calculus reform project purges calculus of its mathematical rigor, resulting in a watered-down version that poorly prepares students for advanced mathematical and scientific training. Calculus is quite literally the language of science and engineering. While the concepts and formalisms of calculus are more than 300 years old, they have never been more centrally important than they are today.Â According to the authors of Assessing Calculus Reform Efforts , over 95% of the colleges and universities that incorporated calculus reform into their curriculums continued to use reform texts the next year, and very few faculty members who have tried the reform approach have gone on to abandon it. Problems and exercises throughout are based on real-life sitÂ uations, and many are similar to questions appearing on the AP* exams. The series chapter uses technology to enhance understanding.Â The course descriptions for the two Advanced Placement courses (Calculus AB and Calculus BC) have changed over the years to respond to new technology and to new points of emphasis in college and university courses. The updated editions of this textbook have consistently responded to those changes to make it easier for students and teachers to adjust.Â Similarly, the coverage of some other topics has been trimmed to reflect the intent of their inclusion in the AP* courses: â€¢ The use of partial fractions for finding antiderivatives has been narrowed to distinct. Epistemological Foundations of Mathematical Experiences, New York: Springer-Verlag, 1991. 19. (with Keith Schwingendorf) Constructing Calculus Concepts: Cooperation in a Computer Laboratory, in Carl Leinbach (ed.) The Laboratory Approach to Teaching Calculus, MAA Notes, 20 1991.Â 62. (with Olga Yiparaki) Predicate Calculus and the Mathematical Thinking of Students. Download as PDF File (218K, 19 pages). 63. (with Despina Stylianou) Determining Linearity: The Interplay between Visualization and Analyses.Â 91. Some General and Specific thoughts about Calculus Reform, University of Georgia Colloquium, February 28, 1997. 92. Putting Constructivism to Work-Spanning the Gap between Theory and Practice, University of No.