Local function spaces

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Isotropic distributional (or Lebesgue-integrable) spaces on $\mathbb{R}^n$.

The Sobolev-Nikol'skij-Besov-Peetre line.
Sobolev: 1936-38, $W_p^k(\mathbb{R}^n)$ (and, mainly, in domains), $D^\alpha f$.
Nikol'skij-Besov: 1951, 1960, $B^s_{p,q}(\mathbb{R}^n) = B^s_q L_p(\mathbb{R}^n)$, $p \geq 1, s > 0$, $\Delta_h^m f$.
Peetre: 1967-75, $A^s_{p,q}(\mathbb{R}^n)$, $0 < p, q \leq \infty, s \in \mathbb{R}$, $\varphi_j(D)f$.

The Morrey-Campanato-Brudnyi line.
Morrey: 1938, $\mathcal{L}^r_p(\mathbb{R}^n)$, $p \geq 1$, $-n/p \leq r < 0$.
Campanato: 1963-65, $\mathcal{L}^r_p(\mathbb{R}^n)$, $-n/p \leq r < \infty$, $p \geq 1$,
Brudnyi: 1965-70, $\mathcal{L}^r_p(\mathbb{R}^n)$, $0 < p < \infty$.

Attempts to merge.
Kozono-Yamazaki: 1994, $B^s_q \mathcal{L}^r_p(\mathbb{R}^n)$, $s \in \mathbb{R}$, $p > 1$, $-n/p \leq r < 0$.
Tang-Xu: 2005, $A^s_q \mathcal{L}^r_p(\mathbb{R}^n)$, $s \in \mathbb{R}$, $0 < p \leq \infty$, $-n/p \leq r < 0$.
Dachun Yang: 2008, $A^{s,\tau}_{p,q}(\mathbb{R}^n)$, $s, \tau \in \mathbb{R}$, $0 < p, q \leq \infty$,
T: 2012, $\mathcal{L}^r A^s_{p,q}(\mathbb{R}^n)$, $s \in \mathbb{R}$, $0 < p, q \leq \infty$, $-n/p \leq r < \infty$.

Comments: Sometimes domain $\Omega \subset \mathbb{R}^n$ instead of $\mathbb{R}^n$ (Sobolev, Brudnyi). Recall $A \in \{B, F\}$. Here preference to $A = B$. Goal is to convince that merging is more than generalising.

Sobolev:

$$\|f\,|W_p^k(\mathbb{R}^n)\| = \sum_{|\alpha|\le k} \|D^\alpha f\,|L_p(\mathbb{R}^n)\|, \quad k \in \mathbb{N}, \quad 1 \le p < \infty.$$

Nikol'skji-Besov:

$$1 \le p \le \infty, \quad 0 < s < m \in \mathbb{N},$$

$$\Delta_h^1 f(x) = f(x+h) - f(x), \qquad \Delta_h^m = \Delta_h^1 \Delta_h^{m-1}, \qquad h \in \mathbb{R}^n, \quad m = 2, 3, \ldots,$$

$$\|f\,|B^s_{p,\infty}(\mathbb{R}^n)\| = \|f\,|L_p(\mathbb{R}^n)\| + \sup_{|h|\le 1} |h|^{-s} \, \|\Delta_h^m f\,|L_p(\mathbb{R}^n)\|.$$

Similarly $B^s_{p,q}(\mathbb{R}^n)$. Special case, Hölder-Zygmund spaces

$$\mathcal{C}^s(\mathbb{R}^n) = B^s_{\infty,\infty}(\mathbb{R}^n),$$

with

$$\|f\,|\mathcal{C}^s(\mathbb{R}^n)\| = \sup_{x\in\mathbb{R}^n} |f(x)| + \sup_{x\in\mathbb{R}^n, |h|\le 1} |h|^{-s} |\Delta_h^m f(x)|.$$

Peetre:

If $\psi \in S(\mathbb{R}^n)$ then $\psi(D)f(x) = (\psi\widehat{f})^\vee(x)$ entire analytic function for any $f \in S'(\mathbb{R}^n)$.

Resolution of unity: $\varphi_0 \in S(\mathbb{R}^n)$, $\varphi = \{\varphi_j\}_{j=0}^\infty$,

$$\varphi_0(x) = 1 \text{ if } |x| \le 1 \quad \varphi_0(y) = 0 \text{ if } |y| \ge 3/2,$$

$$\varphi_k(x) = \varphi_0(2^{-k}x) - \varphi_0(2^{-k+1}x), \qquad x \in \mathbb{R}^n, \quad k \in \mathbb{N}.$$

$$\|f \,|B^s_{p,q}(\mathbb{R}^n)\|_\varphi = \Big(\sum_{j=0}^\infty 2^{jsq} \|\varphi_j(D)f \,|L_p(\mathbb{R}^n)\|^q \Big)^{1/q},$$

Similarly $F^s_{p,q}(\mathbb{R}^n)$, covering Sobolev spaces $H^s_p(\mathbb{R}^n)$, $1 < p < \infty$, $s \in \mathbb{R}$, and classical Sobolev spaces $W^k_p(\mathbb{R}^n) = H^k_p(\mathbb{R}^n)$, $k \in \mathbb{N}_0$.

Q_{JM}, $J \in \mathbb{N}_0$, $M \in \mathbb{Z}^n$, cube in $\mathbb{R}^n$, left corner $2^{-J}M$, sides parallel to the axes, length 2^{-J+1}.

Notation:

uniform space: $\sup_{M \in \mathbb{Z}^n} \cdots Q_{0,M}$, local space: $\sup_{J \in \mathbb{N}_0, M \in \mathbb{Z}^n} \cdots Q_{JM}$.

$\mathcal{L}_p(\mathbb{R}^n)$, $0 < p \leq \infty$, uniform L_p-space:

$$\|f \,|\mathcal{L}_p(\mathbb{R}^n)\| = \sup_{M \in \mathbb{Z}^n} \|f \,|L_p(Q_{0,M})\|.$$

$\mathcal{L}_\infty(\mathbb{R}^n) = L_\infty(\mathbb{R}^n)$. $\mathbb{N}_{-1} = \mathbb{N}_0 \cup \{-1\}$.
$\mathcal{P}_k$: all polynomials of degree $\leq k$ with $k \in \mathbb{N}_{-1}$, where $\mathcal{P}_{-1} = \{0\}$.

Definition 1.1. $0 < p \leq \infty$, $-n/p \leq r < \infty$, $k \in \mathbb{N}_{-1}$, $k + 1 > r$. Then $\mathcal{L}^r_p(\mathbb{R}^n)$: all measurable functions in $\mathbb{R}^n$, such that

$$\|f \,|\mathcal{L}^r_p(\mathbb{R}^n)\|_k = \|f \,|\mathcal{L}_p(\mathbb{R}^n)\| + \sup_{J \in \mathbb{N}, M \in \mathbb{Z}^n} 2^{J(\frac{n}{p}+r)} \inf_{P \in \mathcal{P}_k} \|f - P \,|L_p(Q_{JM})\|$$

is finite.

Theorem 1.2. (i) $\mathcal{L}_p^r(\mathbb{R}^n)$ independent of k (equivalent quasi-norms).
(ii) $0 < p \le \infty$, $r > 0$. Then $\mathcal{L}_p^r(\mathbb{R}^n) = \mathcal{C}^r(\mathbb{R}^n)$.
(iii) $0 < p < \infty$. Then $\mathcal{L}_p^{-n/p}(\mathbb{R}^n) = \mathcal{L}_p(\mathbb{R}^n)$.
(iv) $0 < p < \infty$, $-n/p < r < 0$. Let $t = n/|r|$. Then

$$L_\infty(\mathbb{R}^n) \hookrightarrow \mathcal{L}_t(\mathbb{R}^n) \hookrightarrow \mathcal{L}_{t,\infty}(\mathbb{R}^n) \hookrightarrow \mathcal{L}_p^r(\mathbb{R}^n) \hookrightarrow \mathcal{L}_p(\mathbb{R}^n).$$

All embeddings are *into*, but not *onto*. If $\mathcal{L}_{w,v}(\mathbb{R}^n) \hookrightarrow \mathcal{L}_p^r(\mathbb{R}^n)$ then $\mathcal{L}_{w,v}(\mathbb{R}^n) \hookrightarrow \mathcal{L}_{t,\infty}(\mathbb{R}^n)$.
(v) $1 \le p < \infty$, $-n/p \le r < 0$. Then

$$\mathcal{L}_p^r(\mathbb{R}^n) \hookrightarrow \mathcal{C}^r(\mathbb{R}^n) \cap \mathcal{L}_p(\mathbb{R}^n).$$

(vi)

$$\mathcal{L}_\infty^0(\mathbb{R}^n) = \mathcal{L}_\infty(\mathbb{R}^n) = L_\infty(\mathbb{R}^n),$$
$$\mathcal{L}_p^0(\mathbb{R}^n) = bmo(\mathbb{R}^n), \qquad 0 < p < \infty.$$

Remark 1.3. $r < 0, k = -1$: Morrey 38. $r = 0$: John-Nirenberg, 61, $r > 0$: Campanato 64, Brudnyi, 65-69, 71 ($0 < p < 1$), survey 09, Peetre 69, (vi) Bennett, Sharpley 79.

$\mathcal{L}^r_p(\mathbb{R}^n)$, $0 < p < \infty$, $-n/p \le r < 0$ used for nonlinear PDEs, Navier-Stokes equation: Morrey, Giga-Miyakawa 89, M. Taylor 92. Extension to smooth spaces, including applications: Kozono-Yamazaki 1994, Mazzucato 2003.

$\varphi = \{\varphi_j\}$ above resolution of unity in $\mathbb{R}^n$.

Definition 1.4. $0 < p, q \le \infty$, $s \in \mathbb{R}$, $-n/p \le r < 0$.

$$B^s_q \mathcal{L}^r_p(\mathbb{R}^n) = \{ f \in S'(\mathbb{R}^n) : \ \| f \, | B^s_q \mathcal{L}^r_p(\mathbb{R}^n) \| < \infty \}$$

with

$$\| f \, | B^s_q \mathcal{L}^r_p(\mathbb{R}^n) \| = \Big(\sum_{j=0}^{\infty} 2^{jsq} \| \varphi_j(D) f \, | \mathcal{L}^r_p(\mathbb{R}^n) \|^q \Big)^{1/q}.$$

Similarly $F^s_q \mathcal{L}^r_p(\mathbb{R}^n)$.

Truncation of $\varphi_j(D)f$ and localization: Dachun Yang, Wen Yuan, 08. Several papers afterwards, Sawano, ..., book Yuan, Sickel, Yang, Lecture Notes Math. 2005 (2010)

Definition 1.5. $0 < p, q \leq \infty$ ($p < \infty$ for F-spaces), $s \in \mathbb{R}$, $\tau \geq 0$.

$$\mathcal{B}^{s,\tau}_{p,q}(\mathbb{R}^n) = \{f \in S'(\mathbb{R}^n) : \ \|f \,|\mathcal{B}^{s,\tau}_{p,q}(\mathbb{R}^n)\| < \infty\}$$

with

$$\|f \,|\mathcal{B}^{s,\tau}_{p,q}(\mathbb{R}^n)\| = \sup_{J\in\mathbb{N}_0, M\in\mathbb{Z}^n} 2^{Jn\tau} \Big(\sum_{j=J}^{\infty} 2^{jsq} \|\varphi_j(D)f \,| L_p(Q_{JM})\|^q \Big)^{1/q}.$$

and similarly $\mathcal{F}^{s,\tau}_{p,q}(\mathbb{R}^n)$.

Remark 1.6. Modification of corresponding spaces $A^{s,\tau}_{p,q}(\mathbb{R}^n)$ (all cubes Q_{JM} with $J \in \mathbb{Z}$, $M \in \mathbb{Z}^n$ but φ as above) in literature, also in book Yuan-Sickel-Yang.

Motivation for truncation: $bmo(\mathbb{R}^n)$, John-Nirenberg 61.

$$\|f\,|bmo(\mathbb{R}^n)\| = \sup_{J\in\mathbb{N}_0, M\in\mathbb{Z}^n} 2^{Jn} \int_{Q_{JM}} |f(x) - f_{Q_{JM}}|\,\mathrm{d}x + \sup_{M\in\mathbb{Z}^n} \int_{Q_{0,M}} |f(x)|\,\mathrm{d}x$$

with the mean value

$$f_{Q_{JM}} = |Q_{JM}|^{-1} \int_{Q_{JM}} f(y)\,\mathrm{d}y.$$

$bmo(\mathbb{R}^n) = F^0_{\infty,2}(\mathbb{R}^n)$. Extension to $F^s_{\infty,q}(\mathbb{R}^n)$ with $s \in \mathbb{R}$, $0 < q < \infty$: Frazier-Jawerth 90,

$$\|f\,|F^s_{\infty,q}(\mathbb{R}^n)\| = \sup_{J\in\mathbb{N}_0, M\in\mathbb{Z}^n} \Big(2^{Jn} \int_{Q_{JM}} \sum_{j=J}^{\infty} 2^{jsq} |\varphi_j(D)f(x)|^q\,\mathrm{d}x \Big)^{1/q}$$

Motivation for brutal truncation of the entire analytic functions $\varphi_j(D)f$: points in lattices of mesh-length 2^{-j} determine

$$\varphi_j(D)f(x) = F^{-1}\varphi_j F\,f(x), \qquad x \in \mathbb{R}^n,$$

restriction to cubes Q_{JM} makes if $j \geq J$, measured in $L_\infty(\ell_q)$.

Wavelets $\psi_F \in C^u(\mathbb{R})$, $\psi_M \in C^u(\mathbb{R})$, $u \in \mathbb{N}$,

$$\int_{\mathbb{R}} \psi_M(x) x^v \,\mathrm{d}x = 0, \qquad v \in \mathbb{N}_0, \quad v < u.$$

$$\Psi^j_{G,m}(x) = 2^{jn/2} \prod_{w=1}^{n} \psi_{G_w}(2^j x_w - m_w), \qquad G \in G^j, \quad m \in \mathbb{Z}^n.$$

Q_{jm} cube in $\mathbb{R}^n$, $2^{-j}m$ left corner, side-length 2^{-j+1}, $j \in \mathbb{N}_0$, $m \in \mathbb{Z}^n$,

$$\operatorname{supp} \Psi^j_{G,m} \subset Q_{jm}.$$

Wavelet expansion:

$$f = \sum_{j=0}^{\infty} \sum_{G \in G^j} \sum_{m \in \mathbb{Z}^n} \lambda^{j,G}_m \, 2^{-jn/2} \Psi^j_{G,m},$$

with

$$\lambda^{j,G}_m = \lambda^{j,G}_m(f) = 2^{jn/2} \int_{\mathbb{R}^n} f(x) \Psi^j_{G,m}(x) \,\mathrm{d}x.$$

Wavelets as above,

$$\Psi^u = \left\{ \Psi^j_{G,m} : \ j \in \mathbb{N}_0, \ G \in G^j, \ m \in \mathbb{Z}^n \right\}, \quad \text{based on } \psi_{F,u},$$

$$\mathrm{supp}\ \Psi^j_{G,m} \subset Q_{jm}.$$

$$\mathbb{P}_{JM} = \left\{ j \in \mathbb{N}_0, \ G \in G^j, \ m \in \mathbb{Z}^n : \ Q_{jm} \subset Q_{JM} \right\}, \qquad J \in \mathbb{N}_0, \quad M \in \mathbb{Z}^n.$$

Sequence spaces $\mathcal{L}^r b^s_{p,q}(\mathbb{R}^n)$:

$$\lambda = \left\{ \lambda^{j,G}_m \in \mathbb{C} : \ j \in \mathbb{N}_0, \ G \in G^j, \ m \in \mathbb{Z}^n \right\}$$

quasi-normed by

$$\| \lambda \,| \mathcal{L}^r b^s_{p,q}(\mathbb{R}^n) \| = \sup_{J \in \mathbb{N}_0, M \in \mathbb{Z}^n} 2^{J(\frac{n}{p}+r)} \Big(\sum_{j=J}^{\infty} 2^{j(s-\frac{n}{p})q} \Big(\sum_{m,G:(j,G,m)\in \mathbb{P}_{JM}} |\lambda^{j,G}_m|^p \Big)^{q/p} \Big)^{1/q}.$$

Similarly $\mathcal{L}^r f^s_{p,q}(\mathbb{R}^n)$.

Definition 2.1. $0 < p, q \leq \infty$, $s \in \mathbb{R}$, $-n/p \leq r < \infty$,

$$u > \max(s + r^+, \sigma_p - s).$$

Then $\mathcal{L}^r B^s_{p,q}(\mathbb{R}^n)$ collects all $f \in S'(\mathbb{R}^n)$ for which

$$\|f \,|\mathcal{L}^r B^s_{p,q}(\mathbb{R}^n)\|_{\Psi^u} = \sup_{J,M} 2^{J(\frac{n}{p}+r)} \Big\| \sum_{(j,G,m)\in\mathbb{P}_{JM}} \lambda^{j,G}_m(f)\, 2^{-jn/2} \Psi^j_{G,m} \,|B^s_{p,q}(\mathbb{R}^n)\Big\|$$

$$\sim \|\lambda(f)\,|\mathcal{L}^r b^s_{p,q}(\mathbb{R}^n)\| < \infty.$$

Recall $\sigma_p = n\big(\max(1/p, 1) - 1\big)$.

Usual lift in $S'(\mathbb{R}^n)$: $I_\sigma f = F^{-1}(1+|\xi|^2)^{-\sigma/2} Ff$, $\sigma \in \mathbb{R}$.

$$I_\sigma B^s_{p,q}(\mathbb{R}^n) = B^{s+\sigma}_{p,q}(\mathbb{R}^n).$$

Theorem 2.2. (i) $\mathcal{L}^r B^s_{p,q}(\mathbb{R}^n)$ is independent of Ψ^u.

$$\mathcal{L}^r B^s_{p,q}(\mathbb{R}^n) \hookrightarrow \mathcal{C}^{s+r}(\mathbb{R}^n).$$

$$I_\sigma \mathcal{L}^r B^s_{p,q}(\mathbb{R}^n) = \mathcal{L}^r B^{s+\sigma}_{p,q}(\mathbb{R}^n).$$

(ii) $r > 0$, then

$$\mathcal{L}^r B^s_{p,q}(\mathbb{R}^n) = \mathcal{C}^{s+r}(\mathbb{R}^n).$$

(iii) $r = 0$:

$$\mathcal{L}^0 B^s_{p,\infty}(\mathbb{R}^n) = \mathcal{C}^s(\mathbb{R}^n), \qquad 0 < p \le \infty, \quad s \in \mathbb{R}.$$

$$\mathcal{L}^0 L_p(\mathbb{R}^n) = bmo(\mathbb{R}^n), \qquad 2 \le p < \infty.$$

Remark 2.3. $\mathcal{L}^0 L_2(\mathbb{R}^n) = bmo(\mathbb{R}^n)$: Y. Meyer, ~ 90.

If $r < 0$ then there is an $f \in \mathcal{C}^{s+r}(\mathbb{R}^n)$ with $f \notin \mathcal{L}^r B^s_{p,q}(\mathbb{R}^n)$ for all admitted p, q, A.

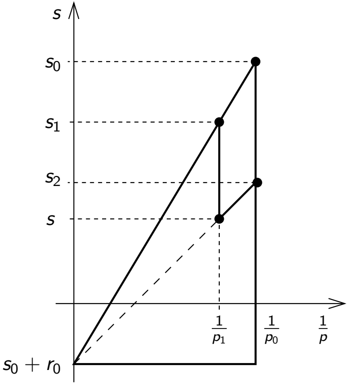

Figure: Limiting embeddings

Main problem: Necessary and sufficient conditions for

$$\mathcal{L}^{r_0} B^{s_0}_{p_0,q_0}(\mathbb{R}^n) \hookrightarrow \mathcal{L}^{r} B^{s}_{p,q}(\mathbb{R}^n)$$

Two decisive quantities:

$$\text{differential dimension}: \quad s + r,$$
$$\text{slope}: \quad |r|p.$$

Classical case: $r = -n/p$. Then differential dimension $s - \frac{n}{p}$, slope n. Now merging parameters: From (s, p, q) to (r, s, p, q) as in Newtonian Mechanics to Special Relativity.

Proposition 3.1. Let $s_0 \in \mathbb{R}$, $0 < p_0 < \infty$, $0 < q_0 \leq \infty$, $-n/p_0 \leq r_0 < 0$. Let

$$p_0 \leq p_1 < \infty, \qquad s_1 = s_0 + r_0\Big(1 - \frac{p_0}{p_1}\Big), \qquad \frac{q_1}{p_1} = \frac{q_0}{p_0}$$

$$s_1 + r_1 = s_0 + r_0 \qquad \text{(invariance of differential dimension)}.$$

Then

$$r_1 p_1 = r_0 p_0 \qquad \text{(invariance of slope)}$$

and

$$\mathcal{L}^{r_0} B^{s_0}_{p_0,q_0}(\mathbb{R}^n) \hookrightarrow \mathcal{L}^{r_1} B^{s_1}_{p_1,q_1}(\mathbb{R}^n).$$

Furthermore

$$\mathcal{L}^{r_0} B^{s_0}_{p_0,q_0}(\mathbb{R}^n) \hookrightarrow \mathcal{C}^{s_0+r_0}(\mathbb{R}^n).$$

Proof based on wavelet representations, above endpoint embeddings and Hölder inequalities. Disturbing point: q-conditions. Next: $q = \infty$.

Theorem 3.2. Let $s_0 \in \mathbb{R}$, $0 < p_0 < \infty$, $-n/p_0 \le r_0 < 0$. Let

$$p_0 \le p < \infty, \qquad s_0 + r_0 < s \le s_0 + r_0\left(1 - \tfrac{p_0}{p}\right)$$

(triangle in Figure). Let

$$r + s = r_0 + s_0 \qquad \text{(invariance of differential dimensions).}$$

Then $-n/p \le r < 0$ and

$$\mathcal{L}^{r_0} B^{s_0}_{p_0,\infty}(\mathbb{R}^n) \hookrightarrow \mathcal{L}^{r} B^{s}_{p,\infty}(\mathbb{R}^n).$$

Proof: Combine above Proposition 3.1 with

$$\mathcal{L}^{r_0} B^{s_0}_{p,q}(\mathbb{R}^n) \hookrightarrow \mathcal{L}^{r_1} B^{s_1}_{p,q}(\mathbb{R}^n), \quad s_0 + r_0 = s_1 + r_1, \quad s_1 < s_0,$$

Dialectical method: Contradictory thesis and anti-thesis resolve at a higher level (Hegel, Marx).
Sobolev and Morrey as a dialectical couple:

Sobolev: Offer smoothness, ask for better integrability,
Morrey: Offer (refined) integrability, aks for better smoothness.

Morrey's refinement of the (uniform) Lebesgue spaces $\mathcal{L}_p(\mathbb{R}^n)$: $0 < p < \infty$, $-n/p \le r < 0$,

$$\|f \,|\mathcal{L}^r_p(\mathbb{R}^n)\| = \sup_{J \in \mathbb{N}_0, M \in \mathbb{Z}^n} 2^{J(r+\frac{n}{p})} \|f \,|L_p(Q_{JM})\|.$$

Recall that

$$\mathcal{L}^r_p(\mathbb{R}^n) = \mathcal{L}^r L_p(\mathbb{R}^n) = \mathcal{L}^r F^0_{p,2}(\mathbb{R}^n), \qquad \text{if } 1 < p < \infty.$$

Main Problem repeated: Necessary and sufficient conditions for

$$\mathcal{L}^{r_0} A^{s_0}_{p_0,q_0}(\mathbb{R}^n) \hookrightarrow \mathcal{L}^r A^s_{p,q}(\mathbb{R}^n)$$

So far, classical case, $r_0 = -n/p_0$, $r = -n/p$, Sickel-T (1995). Now Morrey's refinement of L_p-spaces should be included guided by:

$$s + r \le s_0 + r_0, \quad \text{decreasing differential dimensions},$$
$$|r|p \le |r_0|p_0, \quad \text{decreasing slopes}.$$

In mathematics, a function space is a set of functions between two fixed sets. Often, the domain and/or codomain will have additional structure which is inherited by the function space. For example, the set of functions from any set X into a vector space has a natural vector space structure given by pointwise addition and scalar multiplication. In other scenarios, the function space might inherit a topological or metric structure, hence the name function space. As the underlying function space we use the Hilbert space X = L2(D). We use the splitting u = t Ï• + w, where w â^^ K = {w: â^«D wÏ• dx = 0}. Denote by P, Q the orthogonal projection of X onto K and span[Ï•]. The proof is based on studying the homotopy. Lu+Î»ku+s(g(x,u)â^'h)+(1â^'s)Qu=0. in.Â . Definition 1.1.5 Local Lebesgue spaces. Let. GâŠ‚Rd. Sobolev Space Local Oscillation Finite Dimensional Space John Domain Local Approximation Space. These keywords were added by machine and not by the authors. This process is experimental and the keywords may be updated as the learning algorithm improves.Â Cite this paper as: Bojarski B. (1988) Remarks on local function spaces. In: Cwikel M., Peetre J., Sagher Y., Wallin H. (eds) Function Spaces and Applications. Lecture Notes in Mathematics, vol 1302.