Quantum Mechanics-(2018)

Part -A Rise of Quantum Mechanics

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§1 Recall and discuss

There were several experiments which pointed to inadequacy of the classical thoeries such as mechanics,electromagnetic theory of light, and statistical mechanics. Here we list some of the important experimental facts which had no explanation within the classical theories.

1. Black Body radiation
2. Photoelectric effect
3. Atomic spectra
4. Frank Hertz experiment
5. Compton effect

6. Stern Gerlach Experiment
7. Zeeman splitting of spectral lines in magnetic field
8. Stark effect
9. Wave nature of electrons
10. Alpha decay
11. Beta decay
12. Specific heat of solids and gases

While efforts to explain some of the above experiments played crucial role in the development of quantum theory, Some other experiments provided crucial support for proposals during 1900-1925, before quantum mechanics was born in 1926. Still an explanation of some others had to wait for a fully developed quantum mechanics.

§2 Rise of quantum theory

1. **Planck's theory Black body radiation (1900)**
   In order to explain the black body radiation Planck introduced assumption that the radiation is not emitted continuously but in form of packets of energy.

2. **Einstein's paper on photoelectric effect (1905)**
   Einstein's explanation of photoelectric effect brought in the particle nature back and led to acceptance of dual nature of radiation. A crucial assumption was that energy of a photon of frequency is $E = h\nu$ and that absorption of light can takes place only by absorbing one or more photons, energy from light is not absorbed continuously as was assumed in the classical electromagnetic theory.

3. **Bohr's theory of H atom spectra (1913), Franck Hertz experiment (1914)**
   Bohr assumed that all orbits are not allowed and an electron does not radiate in certain states, called stationary states. To find the allowed orbits and corresponding energies, Bohr assumed that the angular momentum of the electron can only take values equal to integral multiples of Planck's constant $\hbar$. This rule was later reformulated as the Bohr Sommerfeld quantisation rule.

   Discrete nature of atomic energy levels was confirmed by the Franck Hertz experiment.

4. **Wilson Sommerfeld Quantisation (1916)** The quantisation rule

$$\oint pdq = nh$$

generalized the qunatization of angular momentum used by Bohr's and was found useful for wider applications, one example being relativistic corrections to hydrogen atom levels. Notable failure of this rule was the spectrum of He atom.

5. **Zeeman effect (1896-1916)**
Normal Zeeman effect observed (1896), in some atoms, was explained by Lorentz (1897) using classical electromagnetic theory. It came to be called normal Zeeman effect.After Bohr model was published, Sommerfeld and Debye, independently, gave a explanation of the observed pattern within older quantum theory (1916) before quantum mechanics was born. It required the assumption that the component of angular momentum along the magnetic field is quantized.

   The Zeeman pattern in most atoms is very complex and an explanation had to wait for introduction of spin of electron.

6. **Compton Scattering (1923)** The experimental results on scattering of X-rays by atoms firmly established, beyond any doubt, the corpuscular nature of radiation. In addition to establishing the particle picture for light, the energy momentum relation $E = pc$, a prediction of special relativity was beautifully confirmed by Compton scattering data.

7. **Matter waves(1924)** The idea of associating waves with material particles was introduced by de Broglie. This was confirmed by beautiful experiment of Davisson and Germer on electron diffraction (1927). The data of 1921 experiments was already pointing towards effects of diffraction.

8. **Bose Einstein Statistics (1924)** Bose's derivation of black body radiation opened the doors for treatment of systems of identical particles and quantum statistics.

9. **Heisenberg's Matrix mechanics(1925)** While trying to understand intensities of spectral lines of hydrogen spectrum, Heisenberg laid foundations of matrix mechanics. During this investigation he sought to reformulate Sommerfeld quantisation condition

$$\oint pdq = nh$$

and arrived at

$$h = 4\pi nm \sum_{\alpha=0}^{\infty} \left\{ |a(n, n+\alpha|^2 \omega(n+\alpha) - |a(n, n-\alpha)|^2 \omega(n, n-\alpha) \right\}. \tag{1}$$

In this work Heisenberg assumed that the probabilities of transition from n to $n-\alpha$ was proportional to $|a(n, n-\alpha)|^2$. This paper was written in middle of July 1925 and was sent to Max Born. Max Born wrote Heisenberg's equation in the form

$$\sum_k \left[p(n,k)q(k,n) - q(n,k)p(k,nn) = \frac{h}{2\pi i} \right. \tag{2}$$

and realised that the relations was diagonal element of the matrix form of quantum condition

$$pq - qp = \frac{h}{2\pi i} I. \tag{3}$$

To learn more about Eq.(1) *refer to Heisenberg's original paper.* Matrix mechanics was eventually completed by Heisenberg, Born and Jordan by October 1925 and was applied to quantisation of electromagnetic field.

10. **Canonical quantisation(1925)** Dirac brought in the correspondence of matrix commutator in quantum theory with Poisson brackets in classical theory. Fowler had asked Dirac to give his comments on the manuscript of Heisenberg's paper. Dirac went on to develop full abstract machinery of quantum mechanics as we know today. Pauli, who had been initially critical of formal matrix approach, solved the hydorgen atom problem within the matrix mechanics.

11. **Introduction of spin (1925)**
Goudsmith and Uhlenbeck's idea to introduce spin was crucial to understanding Stern Gerlach experiment, anomalous Zeeman effect and the fine structure of atomic spectra. Heisenberg and Jordan completed the solution to the problem of anomalous Zeeman effect within matrix mechanics.

12. **Pauli exclusion principle (1925) and periodic Table**

13. **Quantisation as an eigenvalue problem (1926)** Schrödinger as an attempt developed wave mechanics as an alternative to the Matrix mechanics of Born Jordan Dirac and Heisenberg. He proposed quantisation as an eigenvalue problem and solved the Hydrogen atom problem. Schrodinger went on to prove equivalence of wave mechanics with matrix mechanics, discovered independently by Eckart (1926).

14. **Probability in quantum mechanics (1926)** The probabilistic nature already appeared in Heisenberg's work. Working on anomalous Zeeman effect he assumed that the intensities were proportional to absolute square of Fourier coefficients of $x(t)$. Max Born while formulating quantum scattering, arrived at the probabilistic interpretation of the wave function.

15. **Mathematical foundations (1927)** The works of Dirac on classical correspondence, transformation theory, of Hilbert and of von Neumann brought in the Hilbert spaces into quantum mechanics as its mathematical foundation.

16. **Uncertainty principle (1927)** The fact that one could not assign precise values to canonical conjugate pair of variables is implicitly there in Dirac Jordan transformation theory and Dirac and Jordan were aware of this fact. Heisenberg proceeded to formulate this mathematically and arrived at his famous uncertainty principle. Several issues became clear through works of Rurak, Kennard, Condon, Robertson and others before it was established that the uncertainty in the form we understand today.

17. **Complementarity Principle (1928)** Soon after quantum mechanics was fully developed and equivalence of matrix mechanics and wave mechanics was established, questions of interpretation of quantum mechanics were hotly debated. The complementarity principle played an important role in clarifying issues related to the interpretation of quantum mechanics.

Notes and References

§3 New Concepts Brought in by Quantum Theory A summary

We summarize some important classical concepts which underwent a complete revision after the quantum revolution. We relate these changes to the developments that took place before quantum mechanics was established (1900-1924).This list is aimed to provide a motivation for the postulates of quantum mechanics.

- **Discontinuous nature of physical processes**
  The classical physical processes are perceived as happening continuously. In contrast to the prevailing models, Planck's explanation of black body radiation, Einstein's explanation of photoelectric effect and Bohr model, all assumed that the the process of emission of radiation is discontinuous. Niels Bohr brought in quantum jumps to explain hydrogen atom spectrum.

- **Quantization of dynamical variables**
  In classical theories dynamical variables associated with waves and particles can take continuous values.

  In quantum description, the dynamical variables are quantized, in general, they can take only some discrete values. This was needed for many successful explanations of physical phenomena in days of old quantum theory, Bohr's model being one such example. That all values are not allowed for z- component of angular momentum, was postulated in order to explain Zeeman effect. The Stern Gerlach experiment demonstrated quantization of intrinsic magnetic moment and of spin.

- **Simultaneous measurement**
  Unlike classical theories, in general two arbitrary dynamical variables cannot be measured simultaneously. It has roots in Heisenberg's work that the position and canonical conjugate momentum cannot be measured to arbitrary accuracy simultaneously. Limitations on simultaneous measurement of position and momentum can be traced back to wave particle duality.

- **Wave particle duality** In classical theory we associate a well defined trajectory with motion of particles. Waves are not localized and one cannot associate definite trajectories with waves. Properties of particles and waves are incompatible properties.

- **Complementarity principle**

  In general, electrons, protons etc. are described by wave packets and have position and momentum defined subject to restrictions imposed by the uncertainty principle.

  A particle description becomes a good approximation in certain regimes such as small wavelength. The wave nature takes over for large wavelengths.

  The two natures are complementary. They are both needed to have a complete understanding of physical systems. The two aspects do not manifest themselves in any single experiment. (Bohr complementarity principle)

- **Quantum tunnelling**
  The classical motion of particle is confined to regions where the total energy is greater than the potential energy. A classical particle cannot cross a region where the potential energy is higher than the kinetic energy. (Barrier tunnelling as in alpha decay.)

This does not surprise us if we look at the wave picture. The tunnelling through a barrier can be understood in terms of evanescent waves in a medium where the waves can not propagate.

- **Superposition principle**
  Within the classical framework an understanding of the wave phenomena such as interference, diffraction and polarisation require superposition of waves.

  In quantum theory, the wave particle duality also requires a superposition principle. This is *"superposition of states"* as against superposition of wave amplitudes in classical physics. Read more about it from [1].

- **States of physical system**
  The states of a quantum system are no longer specified by the generalised coordinates and momenta.

- **Dynamical variables as operators**
  The dynamical variables are no longer ordinary functions of canonical variables.

- **Quantization as an eigenvalue problem**
  A measurement of a dynamical variable does not give all possible (classically allowed) values. That allowed values are to be computed by solving eigenvalue problem was brought in by works of Schrödinger.

- **Indeterminacy and probability**
  The classical theories are deterministic, where as the quantum theory is probabilistic, again as a consequence of wave particle duality.

  An analysis of thought experiments reveals the indeterministic nature of quantum theory.

  The quantum theory predicts only probabilities of possible outcomes of experiments of measurements.

  Thus a measurement of a dynamical variable yields only probabilities for different possible outcomes of permissible values.

Not only our understanding of classical concepts require a major shift or a complete change and many new concepts are brought in by the quantum theory. In addition entire mathematical framework needed for description of quantum phenomena changes. While the mathematics prerequisite for classical mechanics for solution of problems is differential equations and partial differential equations, quantum mechanics brought in new mathematics. Several possible approaches with variety of starting points are available, the most commonly used one makes use of Hilbert spaces and probability theory in an essential way.

The above description of changes brought by quantum theory is meant to serve as a warning to young readers that a continued attempt to use classical concepts, without care, will result in loss of understanding and seemingly paradoxical situations. It must be remembered that classical mechanics does not become fully obsolete, and that care is required in using classical picture for a given system. In fact correspondence with classical mechanics is capable of providing useful insights and continues to be an active and fertile research area. We end this section with a quote from Landau and Lifshitz [2]

> Thus quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time it requires this limiting case for its own formulation.

§4 Highlights, Points to Remember

You must remember every thing that is given here.

1. Inadequacies of classical theories
2. Landmarks developments which finally shaped quantum theory
3. Conceptual changes brought in by quantum theory.

It is possible that you may not understand some of the points at this stage. A subject is almost never fully understood in first reading You must keep revisiting this lesson frequently. It is hoped that you will be able to understand all that is written here as we progress.

§5 Recommended for Further Reading

[1] For a student of quantum mechanics a strongly recommended book for details of historical account is :
Max Jammer, *The Conceptual Development of Quantum Mechanics*, McGraw-Hill Book Company New York (1966).

[2] The write up on matrix mechanics is based on J. Mehra's account in his Heisenberg memorial lecture at CERN in 1976. This lecture contains details of Heisenberg's contribution and sequence of events in those fateful years. It gives a fascinating accounts of how Heisenberg, and also Born spent sleepless night, before seeing the light in the morning.

Jagdish Mehra,*The Birth of Quantum Mechanics*, Werner Heisenberg Memorial Lecture delivered at the CERN Colloquium on 30 March 1976, CERN Report 76-10.

[3] A brief account of historical development of mathematical foundations of quantum theory can be found in
Arno Bohm, Haydar Uncu and S. Komy, *A Brief Survey of the Mathematics of Quantum Physics*, Reports on Mathematical Physics **64** (2009) 5-32.

References

[1] P. A. M. Dirac. *Principles of Quantum Mechanics.* Oxford University Press, London, 5th edition, 1960.

[2] L. D. Landau and E.M. Lifshitz. *Quantum Mechanics.* Pergamon Press Ltd., London, 2nd edition, 1965.

They are intended to empower students to become independent learners and adept practitioners of quantummechanics." (Mathematical Reviews, July 2010). â€œZettili provides a second edition of this textbook on quantum mechanics. The material is suitable for two undergraduate semesters and one graduate level semester.â€ (Book News, September 2009). Preface: " Nobody understands quantum mechanics" it is the famous statement of Richard Feynman. He claims that we just compromise with quantum mechanics but unable to understand it. It means that we can solve different quantum problems such as schrodinger wave equation but we have difficulty with the bases of quantum mechanics. The most important challenge is the existence of the " objective [Show full abstract] reality" which our mind knows it obviously. The meaning of the objective reality is that beyond all physics laws and independent of observer, there is a fact wh Father of the Quantum Mechanics. Bohrâ€™s insight into the prevailing model of the atom would lay the groundwork for much of the technological advancements of the 20th century. He demonstrated that around each atomic nucleus, there were several levels of concentric shells filled with electrons and showed that the change of an atom's state was due to electrons moving between shells as they absorbed or radiated energy.Â Trying to untangle this and other problems of quantum mechanics has even given rise to new models of reality, such as the theory of the Multiverse. Moreover, the concept of Complementarity is a subject of intense debate not just in the field of physics but also among philosophers, some of whom accuse Bohr of a "simple-minded positivis[m]." 8B Quantum Mechanics A particle of mass m is conned to a one-dimensional box 0 x a. The potential. V (x) is zero inside the box and innite outside. (a) Find the allowed energies of the particle and the normalised energy eigenstates.Â angular momentum quantum number l. Find also the expectation value of a measurement of LË†3 on the state Ïˆ. Part IB, 2017 List of Questions. 2017. Quantum mechanics permits a rationalization of the classically unexplainable observations just described. Even neglecting the ordinary Coulomb repulsion between electrons, there remains a quantum mechanical tendency for electrons to remain separated. This tendency can be treated within the framework of what is called the Pauli exclusion principle, which states that no two electrons in a system can have the same set of quantum numbers. Practically speaking, this requires higher and higher average kinetic energies for the electrons as the electron density increases.