

There are several writings produced here that will help the reader greatly in evaluating Hamilton's achievements in optics and analysis. His major work of geometrical optics consists of the three parts of "Systems of Rays" and its three supplements. Volume one contained most of this material, although only the index was included for the third part of "Systems of Rays." The publication now of the third part will help us to follow those developments in geometrical optics that led him to the prediction of conical refraction and to dynamical theory.

In two letters to De Morgan of 1858, Hamilton discussed definite integrals and divergent series (February 15th) and the solution of third-order differential equations (July 15th). He arrived at these results through the long and diverse use of differential and integral calculus in his researches. Since the relevant parts of these subjects were well established on the Continent, an examination of Hamilton's results should help us to evaluate his achievements in comparison with those of continental mathematicians.

In addition to the easy access to Hamilton's important mathematical works afforded here, readers will enjoy discovering Hamilton's original thoughts in various places in his short essays and letters: the idea of the sympathy between poetry and science, the relation of arithmetic, metrology, and algebra, the nature of analysis and synthesis, and so on. The letters he wrote in his last years reveal his goal of linking mathematics closely with philosophy. Although these shorter essays contain few new scientific insights, they offer important clues concerning what he sought in his work.

Hankins's biography of 1980 has become an important source for the study and analysis of Hamilton's science. In addition, over the past several decades historians of mathematics have composed detailed specialized studies of his work in dynamics and algebra. The volume under review presents previously unexamined and important materials that shine fresh light on Hamilton and will enable us to read his papers from new viewpoints.

Michiyo Nakane  
*Department of Economics, Seijo University,*  
*Seijo, Setagaya, Tokyo, 157-8511 Japan*  
*E-mail address: michiyo.nakane@nifty.com*

10.1016/S0315-0860(03)00059-4

---

### **From Kant to Hilbert: A Source Book in the Foundations of Mathematics**

Selected and edited by William Ewald. Oxford (Clarendon Press). 1996. Two volumes. xviii + 1340 pp. with a personal name index. \$300 US

According to Joseph Dauben's authoritative bibliography *The History of Mathematics from Antiquity to the Present*, source books are relatively recent in mathematics. The earliest mentioned are by Andreas Speiser (1925) and David E. Smith (1929). Among source books treating only part of mathematics, an eminent example is Jean van Heijenoort's *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931* (1967), which has been well regarded by logicians for decades. Indeed, it served as a model for the source book under review, whose title echoes "From Frege to Gödel" with "From Kant to Hilbert."

Van Heijenoort's source book served Ewald as the origin not merely of his title but also of its basic format. Happily for the reader, Ewald (like van Heijenoort) introduces each selection by describing its

contents and placing it in its historical context. The present book is intended to be used concurrently with van Heijenoort's, which Ewald praises highly (p. 2). He explicitly omits from his source book any selections which appeared in van Heijenoort's, in particular two articles by Hilbert (p. 1087).

Ewald follows tradition by dividing the history of the foundations of mathematics into three parts: the Greek period, the 17th century, and the modern period. He then follows his own views by dividing the last into "two subperiods, which overlap both chronologically and in subject matter" (p. 1). The first is from Kant to Hilbert, and the second from Frege to the present. (In effect, van Heijenoort's book dealt with the second.)

The unit Kant-to-Hilbert is natural, Ewald argues. To do so, he muddies the waters by referring to the first subperiod as "the 19th century" and the second as "the 20th century"—an inaccurate statement since Frege's first foundational work was published in 1870. This division is central, however, to how Ewald characterizes those subperiods, namely that the foundations of mathematics is,

in one sense, . . . a technical discipline within mathematics—"mathematical logic", if logic is understood to include . . . set theory. . . . In the other sense, the foundations of mathematics are the concepts, techniques, and structures that are central to mathematical practice—the elements of which mathematics is composed rather than the groundwork on which it rests. Thus, . . . *saturated models*. . . are foundational in the logical sense, but not the mathematical; *groups* . . . are foundational in the mathematical sense, but not the logical; *sets* and *functions* are foundational in both senses. . . . The twentieth century has tended to pursue foundations in the logical sense, while the nineteenth tended to pursue them in the mathematical. (p. 3).

One might conclude that Ewald's source book emphasizes mathematical foundations, such as group theory, within the 19th century.

Yet one would be wrong. Although Ewald used groups to exemplify a concept which is "foundational in the mathematical sense" and although research on group theory flourished during the 19th century, he includes nothing on groups. Nor, in fact, does he restrict himself to the 19th century. His source book could more accurately have been called "From Berkeley to Bourbaki," rather than the unalliterative "From Kant to Hilbert", since it begins with a 1707 notebook by Bishop Berkeley and ends with Bourbaki's 1948 essay "The Architecture of Mathematics."

After Ewald chose his period (i.e., from Kant to Hilbert), he might have resisted the temptation to go off on a tangent at both ends of his 1300-page source book. If Kant really is the appropriate philosophical beginning for his book, then why include the first 130 pages, devoted to Berkeley, Maclaurin, D'Alembert, and their disputes about the infinitesimal calculus? Ewald's violation of his stated intentions, by starting with Berkeley's attack on Newton's fluxions, suggests that there is a conflict between finding an appropriate philosophical beginning (since Kant was the biggest of the "big boys" in 18th-century philosophy) and an appropriate mathematical beginning (since the struggle over the foundations of the calculus was vastly more important, mathematically speaking, than anything Kant ever wrote).

Likewise, Ewald extended the end of his source book well over 100 pages past Hilbert with selections from Brouwer, Zermelo, Hardy, and Bourbaki. Since Ewald ignores groups, what does he emphasize? After his 18th-century excerpts on the calculus and Kant, Ewald turns to geometry. It plays a major role, providing more than a century of selections from Lambert, Gauss, Clifford, Riemann, Helmholtz, Klein, and Poincaré. Then algebra moves to the fore with Duncan Gregory, De Morgan, Hamilton, C. S. Peirce, Dedekind, and Kronecker. Analysis appears again with Bolzano, Dedekind, Cantor, Borel, and Hardy. Finally, there are selections on logic or set theory by Boole, Peirce, Dedekind, Cantor, Poincaré, Hilbert, Brouwer, and Zermelo.

With this overview of Ewald's book in mind, let us turn back to the question implicitly raised at the beginning of this review: What is the purpose of a source book in the history of mathematics? Clearly part of the answer is that such a book makes accessible to the reader important mathematical documents, usually by mathematicians but occasionally by other mathematically informed people, including philosophers. Should the author of such a source book include material that is widely available? There is a tension between including such material and still leaving space for other writings which are less well known but are important on their own account. Ewald includes some extremely familiar items, such as Riemann's "On the Hypotheses which Lie at the Foundation of Geometry" and an excerpt from Newton's *Principia*. Fortunately Ewald tends to include many items which, though less frequently cited, have real interest, such as Bolzano's "Contributions to a Better Grounded Presentation of Mathematics" (1810) and Gregory's "On the Real Nature of Symbolical Algebra" (1840).

Another part of the answer is that a source book ought to include, if possible, translations of material previously unavailable in English. Here Ewald has done well—as exemplified by Bolzano's "Contributions," Kronecker's "On the Concept of Number" (1887), and several other texts.

A third part of the answer is that unpublished letters or manuscripts ought to be included. In this regard, Ewald printed selections from letters between Cantor and Hilbert, as well as between Hamilton and De Morgan. Though all of these letters were previously in print, they were made more accessible and, in the case of Cantor to Hilbert, translated for the first time into English.

Ewald printed one previously unpublished manuscript, an excerpt from Hilbert's 1920 lectures, "Probleme der mathematischen Logik." Yet Ewald's aim here was to shed light not so much on Hilbert as on Kronecker, since the excerpt was included within the slim section on Kronecker. The reviewer wishes that Ewald had taken the trouble to translate more of Kronecker's writings relating to the foundations of mathematics, as well as some of his correspondence, such as his 1884 letter to Cantor. Kronecker remains an enigmatic and seemingly contradictory figure in the history of mathematics, and here Ewald could have made a very positive contribution.

In evaluating a source book, we should consider the authors it omits as well as those it includes. For the period between Kant and Hilbert, there are four authors on the foundations of mathematics for whose inclusion a strong case can be made: Peano, Russell, Schröder, and Weierstrass. Since, however, Ewald's book must be regarded as a complement to van Heijenoort's and since the latter includes selections from Peano and Russell, should we not omit those two authors from our list? No, we should retain them. For van Heijenoort prints only slender excerpts from Peano and Russell, who deserve much better treatment at his hands and at Ewald's. In the case of Russell, there are many intriguing unpublished manuscripts which would have made handsome choices, e.g., his foundational writings during 1906–1908 which led him to the ramified theory of types. (Fortunately, those manuscripts will appear in Volume 5 of Russell's "Collected Papers.")

Even if we indulgently pass over Ewald's omission of Peano and Russell, we cannot condone the exclusion of Schröder and Weierstrass. Extremely little of Schröder's foundational writings have been translated into English. The case is even stronger for Weierstrass, whose foundational influence no mathematician could ignore. Only a philosopher—never a mathematician—could allow such a staggering omission. An excerpt from Weierstrass's many unpublished lectures, or from his correspondence, would have been a valuable choice.

Let us turn, briefly, to a matter of detail. At times Ewald aids his readers by telling them, in an annotation, where a reference or quotation can be found. Thus Berkeley mentions Plato speaking of the mind as a winged chariot, but gives no reference; Ewald kindly informs us of the relevant dialogue,

*Phaedrus*, and the relevant line. Likewise, Berkeley uses the phrase “doctrine of signs,” and Ewald helpfully tells us that Berkeley is referring to Locke’s *Essay Concerning Human Understanding* (p. 57). Yet on some other occasions Ewald does not supply such a reference, even when there is a quotation within his selection. Thus, on pp. 1252–1253, Hardy repeatedly quotes Hilbert without giving the source for the quotations; Ewald supplies no reference, although the first of these quotations appears earlier in his book, on p. 1153, in his selection of Hilbert’s “Die Grundlegung der elementaren Zahlentheorie.”

To sum up, the mathematical reader will benefit from Ewald’s weighty source book, despite its limitations. We are happy to see it join the growing list of foundational books which have substantial interest for both the historian of mathematics and the philosopher of mathematics.

Gregory H. Moore

*Department of Mathematics, McMaster University,  
Ontario L8S 4K1, Canada*

10.1016/S0315-0860(02)00010-1

### **Clifford Truesdell: Eine wissenschaftliche Biographie des Dichters, Mathematikers und Naturphilosophen**

By Yurie A. Ignatieff and Heike Willig. Aachen (Shaker-Verlag). 1999. ISBN 3-8265-4804-3.

Clifford Truesdell died in his 80th year on January 14, 2000, almost a year after this biography was published. He may have been aware of the authors’ project, but I do not think he read it, or even saw it; one wonders what he would have thought of it.

Wonder is the first and last thing I experience in browsing this “wissenschaftliche Biographie.” It is an extremely strange book. Its path to the reading public is dignified enough: It was financially supported by the Alexander von Humboldt-Stiftung in Bonn, Germany, and published by Shaker-Verlag, Aachen, Germany. So one may assume that the project, if not the final text, was refereed and judged fit for publication. Yet the work transplanted me into such a surreal world of thinking and arguing about science and scientists that I see no other way to a review than reporting on the reasons for my wonder.

The preface sets out the authors’ approach to their subject:

unser Held . . . Emeritus Professor *Clifford Ambrose Truesdell* III, der General der modernen Armee der Mathematiker, Mechaniker, Ingenieure, Physiker und Chemiker, die eine dringende Notwendigkeit der Rationalen Natur- und Technikwissenschaften bewußt geworden sind . . . (vii)

Similarly exalted passages occur throughout the book.

In the preface the authors explain the qualities which enabled them to write the biography: they are versed in “Deutsch, Englisch, Französisch, Italienisch, Latein, Mathematik, Mechanik, Philosophie, Physik, Rhetorik, Scheibkunst, Übersetzung, Wissenschaftsgeschichte und -theorie;” they were instructed in these subjects by teachers who, by their “wunderbaren persönlichen Qualität” of modesty, did not wish to be named in person, in order to leave to the authors “Ruhm und Ehre” (p. vii); they are therefore thanked in anonymity.

The authors combine hagiography with an explicit claim of objectivity and truthfulness, elaborated in the Introduction. There they present their own ideas of the true method of history against the background

Request PDF | On Nov 1, 2000, Tony Crilly published *From Kant to Hilbert: A Sourcebook in the Foundations of Mathematics* by William Ewald | Find, read and cite all the research you need on ResearchGate. In a culmination of a long development, it was seen clearly in the early 1930s that steps of formal computation are also steps of formal deduction as defined by recursion equations and other similar principles of arithmetic. Followers of Kant's doctrine of the synthetic a priori in arithmetic missed by a hair's breadth the proper recursive definition of addition that appeared instead first in a [Show full abstract] book of Hermann Grassmann of 1861. Immanuel Kant's *Critique of Pure Reason* is widely taken to be the starting point of the modern period of mathematics while David Hilbert was the last great mainstream mathematician to pursue important nineteenth century ideas. This two-volume work provides an overview of this important era of mathematical research through a carefully chosen selection of articles. They provide an insight into the foundations of each of the main branches of mathematics - algebra, geometry, number theory, analysis, logic, and set theory - with narratives to show how they are linked. Classic works by Bolzano, ISBN 0 19 850537 X (Oxford University Press). Tony Crilly (a1). (a1). Middlesex University Business School, Hendon NW4 4BT. William Bragg Ewald. This massive two-volume reference presents a comprehensive selection of the most important works on the foundations of mathematics. While the volumes include important forerunners like Berkeley, MacLaurin, and D'Alembert, as well as such followers as Hilbert and Bourbaki, their emphasis is on the mathematical and philosophical developments of the nineteenth century. Besides reproducing reliable English translations of classic works by Bolzano, Riemann, Hamilton, Dedekind, and Poincare, William Ewald also includes selections from Gauss, Cantor, Kronecker, and Zermelo, William Bragg Ewald. Immanuel Kant's *Critique of Pure Reason* is widely taken to be the starting point of the modern period of mathematics while David Hilbert was the last great mainstream mathematician to pursue important nineteenth century ideas. This two-volume work provides an overview of this important era of mathematical research through a carefully chosen selection of articles. They provide an insight into the foundations of each of the main branches of mathematics - algebra, geometry, number theory, analysis, logic, and set theory - with narratives to show how they are linked. The collection is an invaluable source for anyone wishing to gain an understanding of the foundation of modern mathematics.