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Principal Investigator: Yi, Yingfei . **Award ID:** 0708331
Organization: GA Tech Res Corp - GIT
Submitted By:
Yi, Yingfei - Principal Investigator
Title:
Multi-frequency oscillations in biological, electrical, and mechanical systems

Project Participants

Senior Personnel

Post-doc

Graduate Student

Undergraduate Student

Technician, Programmer

Other Participant

Research Experience for Undergraduates

Organizational Partners

Other Collaborators or Contacts

Activities and Findings

Research and Education Activities:
My NSF-supported work focused on multi-frequency phenomena arising in biological, electrical, and mechanical systems, with particular attention paid to quasi-periodic motions, almost automorphic intermittency, and bio-chemical oscillations. The results obtained under the support of this grant are listed in the Publications section.

Findings:
Below is a brief summary of my NSF-supported work on quasi-periodic motions, almost automorphic phenomena, bio-chemical oscillations, and their related complexity, concerning Hamiltonian systems, forced circle flows, and closed chemical reaction systems. Numbers in brackets refer to the Publications section in this report.

1. Hamiltonian systems

* KAM stability for properly degenerate Hamiltonian systems.
A properly degenerate Hamiltonian system is a highly degenerate one whose integrable part involves several time scales and at each time scale the corresponding Hamiltonian only depends on part of the action variables. Such a Hamiltonian system arises frequently in problems of celestial mechanics, for instance, in perturbed Kepler problems like the restricted and non-restricted 3-body problems and spatial lunar problems in which several bodies with very small masses are coupled with two massive bodies.

In [3], we extended the classical KAM stability result for 2-time-scales to the case of arbitrary high order proper degeneracy, in order to treat a much broader class of celestial mechanics applications. By developing a quasi-linear KAM scheme, we showed under the weakest higher order non-degenerate conditions of Bruno-Russmann type that the majority of quasi-periodic, invariant tori associated with the integrable part will persist after the non-integrable perturbation. This actually concludes the KAM metric stability for such a properly degenerate Hamiltonian system.

* Quasi-periodic breathers in Hamiltonian networks.
Quasi-periodic breathers (solutions that are quasi-periodic in time and exponentially localized in space) are among the most important coherent structures in Hamiltonian networks, which have been largely found via numerics and experiments in solid state physics and cell-biology. Although breathers are well-known in Hamiltonian PDEs like sine-Gordon equation and cubic NLS equation, they appear to be rare and non-robust objects in Hamiltonian PDEs. To the contrary, numerical studies of many physical models find that the existence of breathers and quasi-periodic breathers is a general phenomenon in Hamiltonian networks, suggesting that the localization property is due to the discreteness and the nature of nonlinearities rather than disorder.

In [1], we considered a class of Hamiltonian networks of weakly, nonlinear, and long-range coupled anharmonic oscillators arising in biological and physical models with either exponential or power-law coupling potential. We showed that, for any given N-admissible sites of the lattice, there is a positive measure set of quasi-periodic breathers having N-frequencies which are only slightly deformed from the on-site frequencies. This work gives a theoretical justification to the numerical findings in the case of long-range coupling.

* Viscous stability of KAM tori.
For a convex Hamiltonian system, viscosity solutions of the corresponding Hamilton-Jaccobi equation form basic dynamical components. They include KAM tori and Mather sets, and are known to coincide with weak KAM solutions. Restricting to a nearly integrable Hamiltonian system, variational regularity of these solutions is of great importance to the nature of Mather sets, related stability issues, and the mechanism of Arnold diffusion.

In [7], we exam stability of KAM tori among the class of viscosity solutions for a nearly integrable Hamiltonian system by introducing a new notion of viscosity stability and stability. We showed that a KAM torus associated to any rotation vector is viscously stable with the stability index less or equal to one-half, and is precisely one-half if the rotation vector is Diophantine. Sharpness of such stability index is shown along with regularity analysis. This result also gives a uniform description about the Diophantine KAM tori and their general KAM tori nearby.

* Quasi-periodic motions in a periodically forced oscillator with singular potential.
In [10], we considered a periodically forced, singular oscillator in which the potential has subquadratic growth at the infinity and admits a singularity. This type of oscillator often arises in quantum fields theory. The existence of many quasi-periodic solutions is shown. This implies the boundedness of all solutions as well as the stability of the system. Aubry-Mather solutions are also discussed.

2. Forced Circle Flows and Cocycles

* Irregular and complex dynamics in forced circle flows.
Continuous almost periodically forced circle flows are among the simplest non-monotone, multi-frequency dynamical systems. They can be generated from almost periodically forced nonlinear oscillators through integral manifolds reduction in the damped cases and through Mather theory in the damping-free cases. They also naturally arise in 2D almost periodic Floquet theory as well as in climate models. Discrete almost periodically forced circle flows arise in the discretization of nonlinear oscillators and discrete counterparts of linear Schrodinger equations with almost periodic potentials. They have been widely used as models for studying strange, non-chaotic attractors and intermittency phenomena during the transition from order to chaos. Hence the study of these flows is of fundamental importance to the understanding of multi-frequency-driven dynamical irregularities and complexities in non-monotone dynamical systems.

In [2], we made a general study on skew-product circle flows in both continuous and discrete settings, with particular attentions paying to almost periodically forced circle flows. When a circle flow is either discrete in time and unforced (i.e., a circle map) or continuous in time but periodically forced, behaviors of minimal sets are completely characterized by classical theory. The general case involving almost periodic forcing is much more complicated due to the presence of multiple forcing frequencies, the topological complexity of the forcing space, and the possible loss of mean motion property. On one hand, we showed that to some extent behaviors of minimal sets in an almost periodically forced circle flow resemble those of Denjoy sets of circle maps in the sense that they can be almost automorphic, Cantorian, and everywhere non-locally connected. But on the other hand, we showed that almost periodic forcing can lead to significant topological and dynamical complexities on minimal sets which are not covered by the contents of Denjoy theory. For instance, an almost periodically forced circle flow can be positively transitive and its minimal sets can be Li-Yorke chaotic and non-almost automorphic.

* Non-uniform hyperbolicity and related dynamics issues for cocycles.
Cocyles are naturally generated from non-autonomous linear systems in both continuous and discrete setting. Their study closely ties up with spectral theory and various dynamics issues arising in quantum field theory such as reducibility, stability, and hyperbolicity.

In [2], we gave a complete classification of minimal sets for the projective flow associated with an almost periodic, sl(2,R)-valued cocycle. This particularly includes cases when the cocycle is non-uniformly hyperbolic.

In [5], we paid a particular attention to family of quasi-periodic Schrodinger cocyles parametrized by both energy and amplitude of potentials. In the case that the potential is analytic and the forcing frequency is Diophantine, Lyapunov exponents are known to have a positive lower bound independent of energy. We showed that this is generically not true for a fixed irrational frequency within the class of continuous potentials.

3. Closed Chemical Reaction systems

* Reversible Lotka-Volterra model.
Many new discoveries in cell biology have indicated the importance of bio-chemical oscillations and it is well-believed that these oscillations can emerge as the collective dynamic behavior of interacting components in the cell. But little understanding is known about the mechanisms and underlying principles of such oscillations, for which modeling and analysis especially using closed system formalisms are essential.

In [11], we developed a reversible Lotka-Volterra model to model the oscillatory reaction dynamics in a closed isothermal chemical system. This is a three-dimensional, dissipative, singular perturbation to the conservative Lotka-Volterra model, with the free energy serving as a global Lyapunov function. The Second Law of Thermodynamics dictates that the system ultimately reach an equilibrium. Quasi-stationary oscillations are analyzed while free energy of the system serves as a global Lyapunov function of the dissipative dynamics. A natural distinction between regions near and far from equilibrium in terms of the free energy is established. We found that the dynamics is analogous to a mechanical system with time-dependent increasing damping. Near equilibrium, no oscillation is possible as dictated by Onsager reciprocal symmetry relation. We also observed that while free energy decreases in the closed system dynamics, it does not follow the steepest descending path.

* Bio-chemical oscillations and multiscale dynamics.
In [9], we investigate the detail oscillatory chemical dynamics in the reversible Lotka-Volterra model. We showed that there is a natural distinction between oscillatory and non-oscillatory regions in the phase space, that is, while orbits ultimately reach the equilibrium in a non-oscillatory fashion, they exhibit damped, oscillatory behaviors as interesting intermediate dynamics. Besides finding of new mechanisms of biological oscillations, this work has several theoretical novelties: a) As time evolves, a single orbit in such a system can behave conservatively, dissipatively, and monotonically; b) While the long term dynamics of such a system are simple, complexity can occur in finite time intervals. These are phenomena which have not been previously discovered in dynamical systems.

Training and Development:

* Mentoring and Career Development
Three postdocs: Zhenguo Liang, Zhenxin Liu, Shuguan Ji, and four graduate students: Yongfeng Li, Jorge Viveros, Yao Li, Xun Huang, were partially supported by this grant. As a result, Liang made a substantial contribution to weak KAM solutions of Hamiltonian systems and he is now an assistant professor at Fudan University; Liu worked on stochastic chemical reaction models and he is now an assistant professor at Jilin University; and Ji worked on periodic solutions of nonlinear wave equations and he is now an associate professor at Jilin University. Y.-F. Li obtained his Ph.D in Fall 2008 on topics of biochemical oscillations and he is now a postdoctoral fellow at IMA; and Viveros obtained his Ph.D in Winter 2007 on topics of quasi-periodic breathers and he is now an assistant professor at Universidad Autonoma del Estado de Hidalgo. Y. Li began to work on his Ph.D thesis on topics of bio-complexity; and Huang obtained his M.S. degree in Summer 2010.

My other postdocs/graduate students supervision during the NSF support period included two young female mathematicians: Qian Wang, a postdocs under the Canadian NSERC Fellowship and Qiuxia Liu, an exchange Ph.D student under a Chinese scholarship.

I also supervised REU projects for two senior students at Georgia Tech: Ahmet Eser from ISYE and Rachel Lunde from CME, who worked with me on projects concerning modeling of production lines and chemical rate equations respectively. The projects generated notable interests among these students towards the study of dynamics. In particular, Lunde has expressed a clear interest in pursuing her graduate study in the School of Math at Georgia Tech in the area of applied dynamics.

Outreach Activities:

As for my other educational activities during the NSF support period, I gave a 12-hour mini-course at the Shanghai Summer School on Dynamical Systems in 2007, a series of lectures at the 13th Chinese National Summer School in Mathematics in 2008, and a frontier lecture at the the Northeast College Math Teacher's Training Camp in summer 2007. I was the main organizer of the 2nd Summer School on Dynamical Systems held in Changchun, China in 2008, and I served on the international scientific committees of the VII Americas School in Differential

Equations and Nonlinear Analysis, held in Cartagena, Colombia in 2007 and PASI2009 on Differential Equations and Nonlinear Analysis, held in Mexico City, Mexico.

In addition, I served as a chair of thesis committees at USTC, China (1 Masters thesis in 2009, and 2 Ph.D and 1 Masters theses in 2007) and Jilin U., China (14 Masters theses in 2009), as well as member of thesis committees at Nanjing U. China (2 Ph.D theses in 2008) and Jilin U., China (6 Ph.D theses in 2008 and 2 Ph.D theses in 2007).

Journal Publications

[1] J. Geng, J. Viveros, Y. Yi, "Quasi-periodic breathers in Hamiltonian networks of long-range coupling", Physica D, p. 2866-2892, vol. 237, (2008). Published,

[2] W. Huang, Y. Yi, "Almost periodically forced circle flows", J. Funct Analysis, p. 832-902, vol. 257, (2009). Published,

[3] Y. Han, Y. Li, Y. Yi, "Invariant tori for Hamiltonian systems with high order proper degeneracy", Ann. Henri Poincare, p. 1419-1436, vol. 8, (2010). Published,

[4] Z. Liu, W. Huang, Y. Yi, "Random chemical reactions, measure stability, and Fokker-Planck equation", in preparation, p. , vol. , (2010). in preparation,

[5] D. Dou, W. Huang, Y. Yi, "Quasi-periodic and weakly mixing dynamics in Hamiltonian systems", Preprint, p. , vol. , (2010). preprint,

[6] W. Huang, Y. Yi, "On Lyapunov exponents of continuous Schrodinger cocycles over irrational rotations", J. Dynam. Differential Equations, p. , vol. , (2010). Submitted,

[7] Z. Liang, J. Yan, Y. Yi, "Viscous stability of Lagrangian tori", Invent. Math., p. , vol. , (2010). Submitted,

[8] D.-W. Huang, H. Wang, Y. Yi, "Bifurcations in a stochastic cycle model", Int. J. Bifur. Chaos, p. , vol. , (2010). Accepted,

[9] Y.-F. Li, H. Qian, Y. Yi, "Nonlinear oscillations and multiscale dynamics in a closed chemical reaction system", J. Dynam. Differential Equations, p. , vol. , (2010). Accepted,

[10] X. Li, Y. Yi, "Littlewood problem for a singular subquadratic potential", Fields Inst. Commun., p. , vol. , (2010). Accepted,

[11] Y.-F. Li, H. Qian, y. Yi, "Oscillations and multiscale dynamics in a closed chemical reaction system: Second law of thermodynamics and temporal complexity", J. Chem. Phys., p. 154505, vol. 129, (2008). Published,

Books or Other One-time Publications

Web/Internet Site

URL(s):

www.math.gatech.edu/~yi

Description:

The papers that are listed in this report are posted at the URL and all the papers acknowledge NSF support. In addition, many of the papers are posted on other preprint servers, such as the ArXiv, further enhancing the dissemination of research supported by this grant.

Other Specific Products

Contributions

Contributions within Discipline:

1. Conferences

The following is a list of my invited lectures given at international conferences during the support period, under partial support of this grant.

(a) Plenary Lectures/Mini-Courses on Conferences:

* Plenary lecture, 2010 ICMC Summer Meeting in Diff. Eqns, Sao Carlos, Brazil (45 min) Feb 2010

* Plenary lecture, Int. Conf. on Nonl. & Stochastic Dynamics, Chengdu, China (45 min) Jun 2009

* Plenary lecture, CRM Conf. on Stability & Instability in Mech. Sys., Barcelona, Spain (1 hr) Sep 2008

* Frontier lectures, The 13th National Summer School in Mathematics, Hefei, China (2 hrs) Jul 2008

* Plenary lecture, Int. Conf. Diff. Equations Appl. Math. Biology, Le Havre, France (1 hr) Jun 2008

* Plenary lecture, 2008 ICMC Summer Meeting in Diff. Eqns, Sao Carlos, Brazil (45 min) Jan 2008

* Plenary lecture, Int. Conf. on Dynam. & Math. Modeling, Valladolid, Spain (45 min) Sep 2007

* Plenary lecture, Symposium on Dynamical Systems, Beijing, China (1 hr) Aug 2007

* Invited address, Northeast College Math Teacher's Training Camp, Changchun, China (1 hr) Jul 2007

* Plenary lecture, VII Americas School: DENA, Cartagena, Colombia (45 min) Jul 2007

* Mini-course, Shanghai Summer School on Dynam. Sys., Shanghai, China (12 hrs) Jun 2007

(b) Other Invited Lectures on Conferences:

* The 3rd Int. Conf. on Recent Advance in App. Dynam. Sys., Guangzhou, China (40 min) Dec 2009

* LCDS Workshop on Lattice Diff. Equations, Providence, USA (1 hr) Nov 2009

* Americas VIII Conf. on Diff. Equations, Veracruz, Mexico (30 min) Oct 2009

* Int. Workshop on Global Dynam. Beyond Uniform Hyperbolicity, Beijing, China (45 min) Aug 2009

* Int. Workshop on Reaction-Diffusion Models & Math. Biology, Harbin, China (45 min) Jun 2009

* Int. Conf. on Asymptotic Anal. & Infinite dim. Dynam. Sys., Hong Kong (40 min.) Jun 2009

* Workshop on Meas. & Top. Dynam. Sys., Suwon, Korea (30 min) Jun 2009

* CAS Workshop on Dynamical Systems, Beijing, China (1 hr) Dec 2008

* Lorentz Center Workshop on KAM Theory & Appl., Leiden, The Nethelands (45 min) Dec 2008

* Special Session Dyn. Appl. Diff. Eq., AMS Sectional Meeting, Huntsville, US (30 min) Oct 2008

* Int. Conf. on Infinite Dim. Dynam. Sys., Toronto, Canada (40 min) Sep 2008

* Canada-China Conf.: Impact of Climate on Infectious Diseases, Nanjing, China (30 min) May 2008

* Symposium on Dynamical Systems, Hefei, China (40 min) Apr 2008

* The Geometry and Analysis of Dynamical Systems, Raleigh, US (30 min) Feb 2008

* ESI Workshop on Renormalization in Dynamical Systems, Vienna, Austria (1 hr) Oct 2007

* The 2nd National Summer School on Dynamical Systems, Changchun, China (45 min) Aug 2007

2. Colloquia/Seminars

The following is a list of my invited lectures given at Institutions during the support period, under partial support of this grant.

* Seminar, Hua Luo-Geng Lab, Chinese Academic Science, Jan 2010

* Dynamical Systems Seminar, Zhongshan University, Dec 2009

* U50 Public Lecture in Math and Interdiscipline Science, York University, Nov 2009

* Dynamical Systems Seminar, Chinese Academy of Science, Aug 2009

* Dynamical Systems Seminar, Peking University, Dec 2008

* Colloquium, Jilin University, Dec 2008

* Lecture Series on Hamiltonian Systems, Shanghai Jiaotong Univ. (four lectures), Jul 2008

* Dynamical Systems Seminar, Nanjing University, May 2008

* Dynamical Systems Seminar, University of Science and Technology of China, May 2008

* Colloquium, Chern Institute, Nankai University, Apr 2008

* Applied Math Seminar, Yorke University, Apr 2008

* Applied Math Seminar, Tulane University, Feb 2008

* Applied Math Seminar, York University, Dec 2007

3. Conference Organization

During this support period, I was involved in the organization of the following professional conferences.

* Member, Scientific Committee: ICMC Summer Meeting on Differential Equations - 2010 Chapter, Sao Carlos, Brazil, Feb 2010

* Member, Scientific Committee: The 3rd Int. Conf. on Recent Advance in Applied Dynam. Sys., Guangzhou, China, Dec 2009

* Member, Scientific Committee: PASI 2009 & VIII Americas Conference in Differential Equations and Nonlinear Analysis, Mexico City and Veracruz, Mexico, Oct 2009

* Organizer, Workshop on Dynamics and Nonlinear Analysis, Changchun, China, July 2009

* Co-organizer: Fields Institute Conference on Infinite Dimensional Dynamical Systems, Toronto, Canada, Sep 2008

* Member, Scientific Committee: Canada-China Conference on the Impact of Climate Change on Vector-borne and Waterborne Diseases, Nanjing, China, May 2008

* Member, Scientific Committee and Organizing Committee: CMC-Summer Meeting on Differential Equations-2008 Chapter, Sao Carlos, Brazil, Jan 2008

* Co-organizer: CAS Worshop on Dynamical Systems, Beijing, China, Aug 2007

* Co-chair, Organizing Committee: The 2nd National Summer School on Dynamical Systems, Changchun, China, Aug 2007

* Member, International Scientific Committee: VII Americas School in Differential Equations and Nonlinear Analysis, Cartagena, Colombia, Jul 2007

4. Editorial

During the support period, I was involved in the editorial works of the following professional journals.

* Co-editor in Chief: Journal of Dynamics and Differential Equations

* Editor: Proc. Amer. Math. Soc.

* Editor: SIAM DSweb Magazine

* Handing Editor: Journal of Differential Equations

* Associate Editor: Taiwanese Journal of Mathematics

* Associate Editor: Journal of Non-autonomous Dynamics and Applications

* Associate Editor: Journal of Differential Equations and Dynamical Systems

* Editorial Board: Annuals of Differential Equations

* Co-editor for Special Fields Inst. Comm. Volume in honor of G. R. Sell

* Co-editor for Special JDDE Issue in honor of J. K. Hale

* Co-editor for: Special JDDE Issue in honor of Z.-F. Zhang

Contributions to Other Disciplines:

Contributions to Human Resource Development:

Contributions to Resources for Research and Education:

* Curriculum development and graduate ODE text.

I developed and taught a special topics course in Fall 2007 titled 'Topics on dynamical complexity' at both graduate and senior levels, for which lecture notes were written and distributed to students. This course gave an introduction to complex behaviors of dynamical systems, with particular attention paid to those arising in differential equations. Topics included chaos in the sense of Li-Yorke and in the sense of positive entropy; strange attractors in dissipative systems; Cantori, Mather sets, and Arnold diffusion in conservative systems; and dynamics under noise perturbations. Part of the results under the support of this grant was used in this course.

During the NSF support period, I worked on two graduate level books. One on multi-frequency phenomena in dynamical systems and the other one is a textbook on ordinary differential equations. The former is based on my lecture notes of several special topics courses which I taught at Georgia Tech during the past several years, aimed at introducing theory, methods, and modern developments of quasi-periodic, almost periodic, and almost automorphic dynamics, as well as connections between them, to graduate students and interested researchers. The latter was a long term project collaborating with my colleagues Shui-Nee Chow and Jack K. Hale focused on providing a comprehensive graduate ODE book with more emphasis on the modern development of dynamics of differential equations. For the past a few years, we wrote a set of lecture notes as basic materials of this book, which has already been used in graduate ODE courses at Georgia Tech and several other universities and was very well received by students.

Contributions Beyond Science and Engineering:

Conference Proceedings

Categories for which nothing is reported:

Organizational Partners
Any Book
Any Product
Contributions: To Any Other Disciplines
Contributions: To Any Human Resource Development
Contributions: To Any Beyond Science and Engineering
Any Conference

Investigation of harmonic oscillations of mechanical systems. 2 Calculation of parameters of harmonic electrical oscillations. 3 4-3. Investigation of damped oscillations of mathematical pendulum. 4 Calculation of parameters of damping electrical oscillations. 5 4-6. Investigation of driven oscillations in oscillation circuit.Â oscillations, not only mechanical. Any physical quantity makes oscillations if it. repetitively varies in opposite directions near some of its value. A motion of a mechanical system near its equilibrium position, during which. the system passes through an equilibrium position over and over in opposite. directions is being termed mechanical oscillations. A state of a system, left by its own, in which it can stay indefinitely long. TWO-STAGE MECHANICAL OSCILLATOR - PENDULUM-LEVER SYSTEM - A Mechanical Amplifier -. The impossible is often the untried. - Jim Goodwin.Â Figure 2. Difference of the potential during oscillation of the physical pendulum 1 - weightless state in the upper position 2 - culmination of force during the fall in the lower position. Since there is a difference in potential (Figure 2.) between the weightless state (1) and culmination of force (2) during oscillation of the pendulum, the same is true for centrifugal force, which is zero in upper position, and culminates in the lower position at maximum speed. Physical pendulum is used as a single-stage oscillator in the system with a lever. Multi-Wave Oscillators. Oscillators can be also be classified into various types depending on the parameter considered i.e. based on the feedback mechanism, the shape of the output waveform, etc.. These classifications types have been given belowÂ Oscillators are a cheap and easy way to generate specific Frequency of a signal.Â Electrical4U is dedicated to the teaching and sharing of all things related to electrical and electronics engineering. Related Articles. Tuned Collector Oscillator. Experimental diagnostics of multi-frequency quasiperiodic oscillations. N.V. Stankevich, A.P. Kuznetsov, E.S. Popova and E.P. Seleznev. June 30, 2016.Â For two-frequency quasi-periodicity this technique is quite suitable, but for higher-dimensional oscillations Fourier spectrum becomes more complex due to a big number of harmonics. Another technique which arises from theory of dynamical chaos is the analysis of the spectrum of Lyapunov exponents. These exponents char-acterize the presence (absence) of the divergence of close phase trajectories [17]. Thus, for a system exhibiting chaotic oscillations, at least one Lyapunov exponent in the spectrum is positive. Thus, for a system exhibiting chaotic oscillations, at least one Lyapunov exponent in the spectrum is positive. For quasi-. periodic oscillations, several Lyapunov exponents become zero, and the number of zero Lyapunov exponents corresponds.Â sider the system of ï¬ve coupled oscillators. Each oscillator is characterized by two phase variables and one frequency of.Â Synchronization and multi-frequency oscillations in the low-dimensional chain of the self-oscil-. lators. Physica D 2013;244(1):36-49 .